Solving Absolute Value Equations

|2x - 3| -4 = 3
|2x - 3| = 7
2x - 3=7 2x – 3 = -7
2x =10 2x = 4
x = 5 x ≠ 2

1. Isolate the absolute value
   - 2. If absolute value equals a positive #, drop the bars and create 2 equations. $|ax+b|$ = c
     - 3. Solve: $ax+b=c$ and 4. Check.
     - 3. Solve: $ax+b=-c$ and 4. Check.
   - 2. If absolute value equals a negative #...
     - 3. No solution

Solving Absolute Value Inequalities

Remember: Anytime you x or ÷ by a negative, flip the inequality symbol.

|2x - 3| -4 ≤ 3
|2x - 3| ≤ 7
2x – 3 ≤ 7 2x – 3 ≥ -7
2x ≤10 2x ≥ 4
x ≤ 5 x ≥ 2
[2, 5]

1. Isolate the absolute value
   - 2. If inequality seems possible, drop the bars and create 2 inequalities. $|ax+b|$ > c
     - 3. Solve: $ax+b>c$ and 4. Check.
     - 3. Solve: $ax+b<-c$ and 4. Check..
   - 2. If absolute value > a negative #...
     - 3. All real solutions
   - 2. If absolute value < a negative #...
     - 3. No solution

Graph your solution set on a number line or express in interval notation.

Evaluating a Piecewise Function

Function notation f(#) means find y when x is that #.
To do this, you will either…

A) substitute x into the function's algebraic expression

B) determine the y coordinate form the plotted ordered pair on the graph at that x.

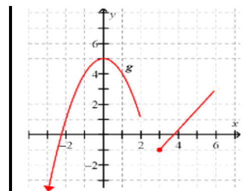
Example:

$$f(x)=\begin{cases} -4x+3 & \text{if } x<3 \\ -x^3 & \text{if } 3\le x\le 8 \\ 3x^2+1 & \text{if } x>8 \end{cases}$$

a.) f(-5) $-4(-5)+3=23$

b.) f(12) $3(12)^2+1=433$

c.) f(4) $-4^3=-64$



g(3) = -1

Transforming a Function

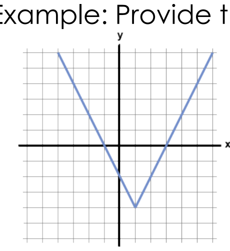
For the following functions:

$y=a(x-h)^2+k$ $y=a|x-h|+k$
$y=a(b)^{x-h}+k$ $y=a\,log_b(x-h)+k$
$y=a\sin b(x-h)+k$ $y=a\cos b(x-h)+k$

the following rules apply:

- a is negative …reflect over x-axis
- |a| > 0…vertical stretch by factor a
- |a| < 0…vertical compression
- h>0 …shift right h units
- h<0…shift left h units
- k>0…shift up k units
- K<0….shift down k units

Example: Provide the equation of the provided graph…

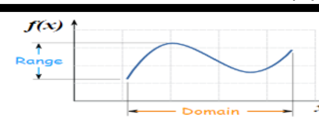


Observations:
Absolute value function, Right 1, Down 4, Stretched such that slope on right is 2.

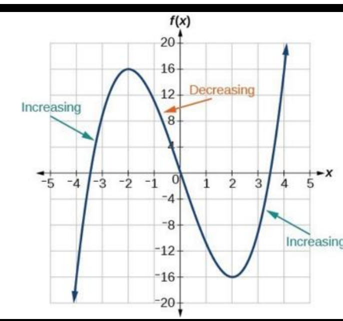
Answer:
F(x) = 2 | x - 1 | - 4

Parts of a Graph

Example responses in (parentheses).



Domain: the set of all possible inputs (x values) of the function. (-∞,∞)
Range: the set of all possible outputs (y values) of the function. (-∞,∞)
Relative/Local Maximum: the highest y value compared to immediate surroundings. Occurs at (-2,16).
Relative/Local Minimum: the lowest y value compared to immediate surroundings. Occurs at (2,-16).
Absolute Maximum/Minimum: the highest/lowest y values reached by the graph. Min. (-∞) and Max. (∞)
X-Intercepts/solutions/real roots: x values where the graph crosses the x-axis (in set notation). {-3.5, 0, 3.5}
Y-intercepts: y values where the graph crosses the y-axis {0}
The function $f(x)=x^3-12x$ is increasing on $(-\infty,-2)\cup(2,\infty)$ and is decreasing on $(-2,2)$.



End Behavior

A function's end behavior (arrow directions) can be identified based on the leading coefficient and degree of the polynomial.

Degree is even and Coefficient is positive... Up&Up → As x→ -∞, f(x) → +∞. As x→ +∞, f(x) → +∞.
Degree is even and Coefficient is negative…Down&Down → As x→ -∞, f(x) → -∞. As x→ +∞, f(x) → -∞.
Degree is odd and Coefficient is positive…Down&Up → As x→ -∞, f(x) → -∞. As x→ +∞, f(x) → +∞.
Degree is odd and Coefficient is negative…Up&Down → As x→ -∞, f(x) → +∞. As x→ +∞, f(x) → -∞.

Example:

$f(x) = 2x^4 - 3x^3 + 5x^2 - 7x + 1$
$f(x) = -2x^4 - 3x^3 + 5x^2 - 7x + 1$
$f(x) = 3x^3 + 5x^2 - 7x + 1$
$f(x) = - 3x^3 + 5x^2 - 7x + 1$

Inverse Function

– a reflection of the function over y = x

1. Switch x and y (or f(x)).
2. Solve for y.
3. Replace y with inverse notation: $f^{-1}(x)$

Note: A relation is one-to-one (1:1) if both the original relation and its inverses are functions.
(should pass both the vertical and horizontal line test)

Example:

$f(x)=\frac{x-7}{x}$, $x\neq 0$ →

$y=\frac{x-7}{x}$
$x=\frac{y-7}{y}$
$xy=y-7$
$xy-y=-7$
$y(x-1)=-7$
$y=\frac{-7}{x-1}$ or $y=\frac{7}{1-x}$

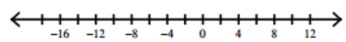
$f^{-1}(x)=\frac{7}{1-x}$; Domain : $x\neq 1$, Range : $y\neq 0$

Unit 1 Practice **Name:** ____________________

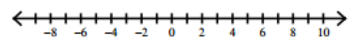
1. $|4x+4| = 28$

2. $|x+2| = -8$

3. $-2|-2p-8| \geq -56$



4. $-1 + |3-2x| < 14$



5. $f(x) = \begin{cases} -2|x+1|, x \leq 1 \\ 3, 1 < x < 3 \\ 6-2x, x \geq 3 \end{cases}$

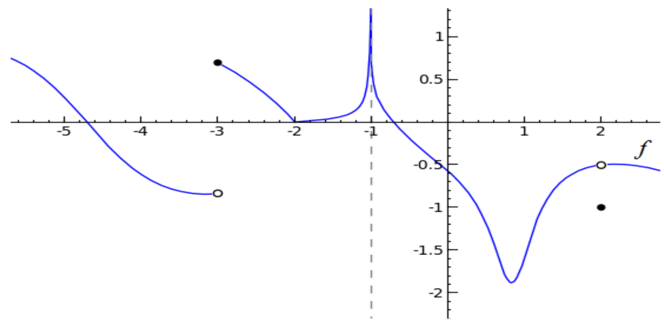
$f(10) =$

$f(0) =$

6. $g(x) = \begin{cases} -2x-1, x \leq 1 \\ -x^2+3x-5, x > 1 \end{cases}$

$g(-2) =$

$g(3) =$



7. f(-3) =

8. f(0) =

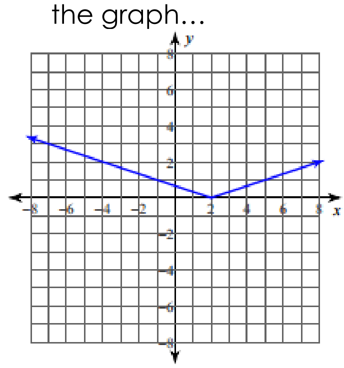
9. Describe the transformations of …

$y = -2(x+3)^2 + 5$

10. Describe the transformations of …

$f(x) = 2^{x+1} - 3$

11. Write the equation of the graph…



12. Write the function that would have the following transformations from

$f(x) = \sqrt{x}$

compress vertically by a factor of 3
reflect across the x-axis
translate right 2 units
translate down 3 units

Use the graph from numbers 7 and 8 to describe the following…

13. Domain: Range:

14. Increasing Interval: Decreasing Interval:

15. Relative Minimum: Absolute Maximum:

16. Describe the end behavior of …

$y = -\frac{4}{3}x^3 + 17x^2 - 5x + 2$

17. Find the inverse of…

$f(d) = 3(d+9)^2$

18. Find the inverse of …

$y = \log_6(4x+4)$

19. Find $f^{-1}(-4)$ if $f(x) = \frac{x-5}{2x+1}$.

20. Graph the inverse for each relation below (put your answer on the same graph).