Objective: Build polynomials from roots and explore the Conjugate Root Theorem.

Notes:

Steps to building polynomials with only real roots:

1. Set x equal to each root
2. Bring the number to the side of the equation with the x
3. Foil the binomials together

Steps to building polynomials with real and complex roots:

1. Follow the previous steps for the real roots and foil these together first
2. Set x equal to the imaginary roots
3. Square both sides
4. Bring the number over to the same side of the equation as x
5. Foil the binomials that came from the imaginary roots together
6. Finally, foil these binomials to the binomials that came from the real roots

The Conjugate Root Theorem states that any time we have an imaginary root we also have its conjugate. (Ex. If 2i is listed as a root, then -2i is also a root.)

Examples:

1) What is the conjugate for each?

- 2+2i $2-2i$
- 3i $-3i$
- -9i $9i$
- √3 $-\sqrt{3}$
- 1+√5 $1-\sqrt{5}$
- 2-√7 $2+\sqrt{7}$

2) Build the polynomial with roots; 2, 3, 4

$x = 2 \quad x = 3 \quad x = 4$

$(x-2)(x-3)(x-4)$

$(x-2)(x^2 - 4x - 3x + 12)$

$(x-2)(x^2 - 7x + 12)$

$x^3 - 7x^2 + 12x - 2x^2 + 14x$
-2

$\boxed{x^3 - 9x^2 + 26x - 24}$

3) Build the polynomial with roots; 2i, 3i

$x^2 = \pm 2i^2 \quad x^2 = \pm 3i^2$

$x^2 = -4 \quad x^2 = -9$

$(x^2+4)(x^2+9)$

$x^4 + 9x^2 + 4x^2 + 36$

$\boxed{x^4 + 13x^2 + 36}$

4) Build the polynomial with roots; 2, 3, -2i

$x = 2 \quad x = 3 \quad x^2 = \pm 2i^2$

$(x-2)(x-3) \quad x^2 = -4$

$(x^2 - 5x + 6) \quad (x^2 + 4)$

$\boxed{x^4 - 5x^3 + 10x^2 - 20x +}$

5) You try!
Build the polynomial with roots; 4, 5, -2

$(x-4)(x-5)(x+2)$

$\boxed{x^3 - 7x^2 + 2x + 40}$

6) Build the polynomial with roots; 4, 4i

$(x-4) \quad x^2 = \pm 4i^2$

$x^2 = -16$

$(x-4)(x^2+16)$

$x^3 + 16x - 4x^2 - 64$

$\boxed{x^3 - 4x^2 + 16x - 64}$

***Keep in mind the number of roots you have is the number of the highest exponent in your equation!**

Name: ______________________

NC Math 3 Unit: ____ Day: ____

Objective: Explore the Fundamental Theorem of Algebra and Multiplicity.

Notes:

Fundamental Theorem of Algebra states that the highest exponent is the number of (real and imaginary) roots the equation has. Remember that imaginary roots will not show up on the calculator, but real roots will show up as x-intercepts.

1. Always graph the equation first
2. Determine how many zeros there are total
3. Place the real root (from the graph) into the box and go through synthetic
4. Rewrite the equation at the end and use the quadratic formula to find the remaining roots

Multiplicity is when the same number is a solution multiple times [ex. (x+2)(x+2)].On a graph, there is multiplicity if the graph hits the x-axis at a max or a min on that number.

Examples:

1) How many zeros does this equation have? What are the real solutions of the equation?

$$2x^3 - 5x^2 = 3x$$

$-\frac{1}{2}, 0, 3$

2) How many zeros does this equation have? What are the real solutions of the equation?

$$x^4 - 8x^2 = -16$$

$-2, -2, 2, 2$

*Multiplicity

3) What are the roots?

$$x(x^2 + 8) = 8(x + 1)$$

$x^3 - 8 = 0$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & 0 & -8 \\ & \downarrow & 2 & 4 & 8 \\ \hline & 1 & 2 & 4 & 0 \end{array}$$

$x^2 + 2x + 4$

$\frac{-2 \pm \sqrt{4-16}}{2} = \frac{-2 \pm 2i\sqrt{3}}{2} = \boxed{-1 \pm i\sqrt{3}}$

4) Solve:

$$x^3 + 2x^2 + 3x + 6 = 0$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 3 & 6 \\ & \downarrow & -2 & 0 & -6 \\ \hline & 1 & 0 & 3 & 0 \end{array}$$

$x^2 + 3 = 0$

$\sqrt{x^2} = \sqrt{-3}$

$\boxed{x = \pm i\sqrt{3}}$

5) $$2x^4 - 12x^3 + 21x^2 + 2x = 33$$

$2x^4 - 12x^3 + 21x^2 + 2x - 33 = 0$

$$\begin{array}{r|rrrrr} -1 & 2 & -12 & 21 & 2 & -33 \\ & \downarrow & -2 & 14 & -35 & 33 \\ \hline & 2 & -14 & 35 & -33 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 2 & -14 & 35 & -33 \\ & \downarrow & 6 & -24 & 33 \\ \hline & 2 & -8 & 11 & 0 \end{array}$$

$\frac{8 \pm \sqrt{(-8)^2 - 4(2)(11)}}{2(2)} =$

$\frac{8 \pm \sqrt{-24}}{4}$ $-24 < {}^{12}_{2} < {}^{6}_{2} < {}^{3}_{2}$

$\frac{8 \pm 2i\sqrt{6}}{4} =$

6) You try!

$$x^3 - 3x^2 + 4x - 12 = 0$$

$$\begin{array}{r|rrrr} 3 & 1 & -3 & 4 & -12 \\ & \downarrow & 3 & 0 & 12 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

$x^2 + 4 = 0$

$\sqrt{x^2} = \sqrt{-4}$

$\boxed{x = \pm 2i}$