

Objective: Solve absolute value inequalities.

Solve each absolute value equation.

Foundation:

$$|x-10|=6$$

$$x-10=6 \quad x-10=-6$$

$$x=16 \quad x=4$$

✓ ✓

$$x = \{4, 16\}$$

$$|4x+6|+8=3$$

$$|4x+6|=-5$$

STOP

No solution!

$$x = \{ \}$$

$$|3x+2|=4x+5$$

$$3x+2=4x+5 \quad 3x+2=-4x-5$$

$$-3 \neq x \quad 7x=-7$$

$$x=-1$$

✓

$$x = \{-1\}$$

$$5-8|-2x|=-75$$

$$-8|-2x|=-80$$

$$|-2x|=10$$

$$-2x=10 \quad -2x=-10$$

$$x=-5 \quad x=5$$

✓ ✓

$$x = \{-5, 5\}$$

$$3-|8x-6|=3$$

$$-|8x-6|=0$$

$$|8x-6|=0$$

$$8x-6=0$$

$$8x=6$$

$$x=3/4$$

✓

$$x = \{3/4\}$$

$$6|1-5x|-9=57$$

$$6|1-5x|=66$$

$$|1-5x|=11$$

$$1-5x=11 \quad 1-5x=-11$$

$$-5x=10 \quad -5x=-12$$

$$x=-2 \quad x=12/5$$

✓ ✓

$$x = \{-2, 12/5\}$$

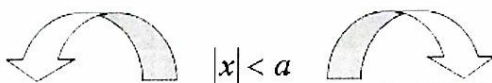
Notes and Examples:

How to solve absolute value inequalities:

Step 1: Isolate the absolute value.

Step 2: Use the definition of absolute value to set up two inequalities:

To set up the two cases:



$x < a$

Case 1: Write the problem without the absolute value sign, and solve the inequality.

$x > -a$

Case 2: Write the problem without the absolute value sign, reverse the inequality, negate the value NOT under the absolute value, and solve the inequality.

Special Cases:

An absolute value cannot be less than a negative number!
No solution!
{0}

$|2x+7| < -12$
An absolute value will always be more than a negative number!

All real numbers
 $(-\infty, \infty)$
 $|x+4| > -7$

Step 3: Solve each inequality from step 2.

Step 4: Use your calculator as visualizer when checking.

Teacher Examples:

1. $|2x+1| > 4$

$$2x+1 > 4 \quad 2x+1 < -4$$

$$2x > 3 \quad 2x < -5$$

$$x > 3/2 \quad x < -5/2$$

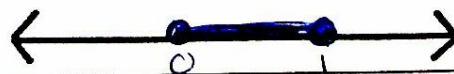
2. $|2x-1|+3 \leq 4$

$$|2x-1| \leq 1$$

$$2x-1 \leq 1 \quad 2x-1 \geq -1$$

$$2x \leq 2 \quad 2x \geq 0$$

$$x \leq 1 \quad x \geq 0$$



1. $|x+4| \leq 8$

$x+4 \leq 8$

$x \leq 4$

$x+4 \geq -8$

$x \geq -12$



2. $|5x| \leq 10$

$5x \leq 10$

$x \leq 2$

$5x \geq -10$

$x \geq -2$



3. $|x+5|-6 < -5$

$|x+5| < 1$

$x+5 < 1$

$x < -4$

$x+5 > -1$

$x > -6$



4. $|6+9x| \leq 24$

$6+9x \leq 24$

$9x \leq 18$

$x \leq 2$

$6+9x \geq -24$

$9x \geq -30$

$x \geq -10/3$



5. $9|x+8|+10 < 55$

$9|x+8| < 45$

$|x+8| < 5$

$x+8 < 5$

$x < -3$

$x+8 > -5$

$x > -13$



6. $|-8x-3| > 11$

$-8x-3 > 11$

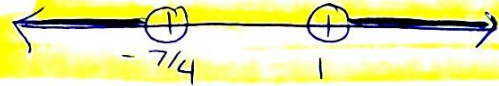
$-8x > 14$

$x < -7/4$

$-8x-3 < -11$

$-8x < -8$

$x > 1$



7. $9|x-8|-10 < 26$

$9|x-8| < 36$

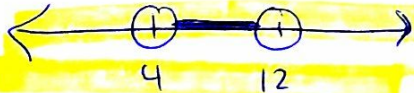
$|x-8| < 4$

$x-8 < 4$

$x < 12$

$x-8 > -4$

$x > 4$



8. $3+4|3x+7| \geq -89$

$4|3x+7| \geq -92$

$|3x+7| \geq -23$

↳ always true

all real numbers (\mathbb{R})

