

Objective: Solve absolute value equations.

Solve the following linear equations:

Foundation:

1. $6x + 7 = 8x - 13$

$20 = 2x$
 $10 = x$

2. $\frac{3}{2}y - y = 4 + \frac{1}{2}y$

$3y - 2y = 8 + y$
 $y = 8 + y$
 $0 = 8$

no solution

3. $\frac{3n-2}{5} = \frac{7}{10}$

$\frac{6n-4}{10} = \frac{7}{10}$
 $6n-4 = 7$
 $6n = 11$

$n = \frac{11}{6}$ or ≈ 1.8

4. $-7(x-3) = -4$

$x-3 = \frac{4}{7}$
 $x = \frac{4}{7} + 3$
 $x = \frac{4}{7} + \frac{21}{7}$
 $x = \frac{25}{7}$ or ≈ 3.6

5. $7-3x = x-4(2+x)$

$7-4x = -4(2+x)$
 $7-4x = -8-4x$
 $7 = -8$

no solution

6. $-8(4+9x) = 7(-2-11x)$

$-32-72x = -14-77x$
 $5x = 18$
 $x = \frac{18}{5}$ or 3.6

How to solve an absolute value equation:

Notes:

Step 1: Isolate the absolute value.

Step 2: Use the definition of absolute value to set up two equations:

If **d** is **POSITIVE** and $|x| = d$, then
 $x = d$ OR $x = -d$
(two equations are set up)

Special Case:

If **d** is **NEGATIVE** and $|x| = d$, then
No solution
This is because distance (**d**) can not be negative.

Step 3: Solve each equation from step 2.

Step 4: Check!!! Extraneous solutions are common.

$|4x+6|+8=3$

$|4x+6| = -5$ **NO!**

Examples:

1. $|3x+2| = 4x+5$

$3x+2 = 4x+5$ $3x+2 = -4x-5$
 $-3 = x$ $7x = -7$
 $x = -1$

$|3(-3)+2| = 4(-3)+5$

$|-7| \neq -7$

$|3(-1)+2| = 4(-1)+5$

$|-1| = 1$ ✓

$x = \{-1\}$

2. $\frac{3}{2}|2x-10| - 11 = -3$

$\frac{2}{3}|2x-10| = 8$
 $\frac{3}{2} \cdot \frac{2}{3}|2x-10| = 8 \cdot \frac{3}{2}$

$|2x-10| = 12$

$2x-10 = 12$

$2x = 22$

$x = 11$

✓

$2x-10 = -12$

$2x = -2$

$x = -1$

✓

$x = \{-1, 11\}$

1. $|5x - 1| = -3x + 15$

$5x - 1 = -3x + 15$

$8x = 16$

$x = 2$

✓

$5x - 1 = 3x - 15$

$2x = -14$

$x = -7$

✓

$x = \{-7, 2\}$

2. $2|-5x - 3| = 10x + 14$

$|-5x - 3| = 5x + 7$

$-5x - 3 = 5x + 7$

$-10 = 10x$

$-1 = x$

✓

$~~-5x - 3 = -5x - 7~~$

$~~-3 = -7~~$

No solution

$x = \{-1\}$

3. $-4|3 - x| = 28$

$|3 - x| = -7$ STOP

$3 - x = -7$

$3 - x = 7$

$-x = -10$

$-x = 4$

$x \neq 10$

$x \neq -4$

$x = \{3\}$

4. $|2x + 4| = 28 + 6x$

$2x + 4 = 28 + 6x$

$2x + 4 = -28 - 6x$

$-24 = 4x$

$8x = -32$

$-6 \neq x$

$x = -4$

✓

$x = \{-4\}$

5. $5|3x - 6| + 9 = 144$

$5|3x - 6| = 135$

$|3x - 6| = 27$

$3x - 6 = 27$

$3x - 6 = -27$

$3x = 33$

$3x = -21$

$x = 11$

$x = -7$

✓

✓

$x = \{-7, 11\}$

6. $|-7x + 4| = 18$

$-7x + 4 = 18$

$-7x + 4 = -18$

$-7x = 14$

$-7x = -22$

$x = -2$

$x = \frac{22}{7}$

✓

✓

$x = \{-2, \frac{22}{7}\}$

7. $|1 - 6n| + 3 = 46$

$|1 - 6n| = 43$

$1 - 6n = 43$

$1 - 6n = -43$

$-6n = 42$

$-6n = -44$

$n = -7$

$n = \frac{22}{3}$

✓

✓

$n = \{-7, \frac{22}{3}\}$

8. $\frac{5|6x - 15| + 7}{11} - 8 = -6$

$\frac{5|6x - 15| + 7}{11} = 2$

11

$5|6x - 15| + 7 = 22$

$5|6x - 15| = 15$

$|6x - 15| = 3$

$6x - 15 = 3$

$6x - 15 = -3$

$6x = 18$

$6x = 12$

$x = 3$ ✓

$x = 2$ ✓

$x = \{2, 3\}$