Name: Key NC Math 3 Unit: 0 Day: 3

Objective: Complete the square to find the vertex form of a quadratic function.

Foundation:

Solve the following equations using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) $m^2 - 5m - 14 = 0$

$m = \frac{5 \pm \sqrt{25 - 4(1)(14)}}{2(1)}$

$m = \frac{5 \pm \sqrt{-31}}{2} = \frac{5 \pm i\sqrt{31}}{2}$

2) $b^2 - 4b + 4 = 0$

$b = \frac{4 \pm \sqrt{16 - 4(1)(4)}}{2(1)}$

$b = \frac{4 \pm \sqrt{0}}{2} = \frac{4 \pm 0}{2}$

$b = 2$

3) $2m^2 + 2m - 12 = 0$

$m^2 + m - 6 = 0$

$m = \frac{-1 \pm \sqrt{1 - 4(1)(-6)}}{2(1)}$

$m = \frac{-1 \pm \sqrt{25}}{2} = \frac{-1 \pm 5}{2}$

$m = \{-3, 2\}$

4) $2x^2 - 3x - 5 = 0$

$x = \frac{3 \pm \sqrt{9 - 4(2)(-5)}}{2(2)}$

$x = \frac{3 \pm \sqrt{49}}{4} = \frac{3 \pm 7}{4}$

$x = \{-1, \frac{5}{2}\}$

5) $x^2 + 4x + 3 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(1)(3)}}{2(1)}$

$x = \frac{-4 \pm \sqrt{4}}{2} = \frac{-4 \pm 2}{2}$

$x = \{-3, -1\}$

6) $2x^2 + 3x - 20 = 0$

$x = \frac{-3 \pm \sqrt{9 - 4(2)(-20)}}{2(2)}$

$x = \frac{-3 \pm \sqrt{169}}{4} = \frac{-3 \pm 13}{4}$

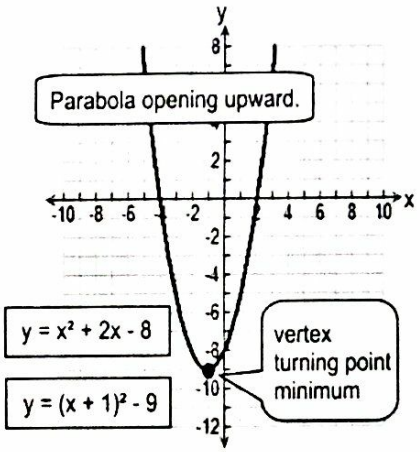
$x = \{-4, \frac{5}{2}\}$

Notes and Examples:

Completing the Square to Find Vertex Form:

1. Examine the graph at the right. Using the process of completing the square, a quadratic function can be expressed in $y = a(x - h)^2 + k$ form, where (h,k) is the vertex of the parabola. This form is often referred to as the "vertex form" since it makes the turning point easily readable.

   What are the coordinates of the vertex of the parabola $y = x^2 + 2x - 8$? $(-1, -9)$



2. Using the process of completing the square, write the following quadratic functions in "vertex form", state the coordinates of the vertex (turning point), and indicate opening direction as up or down:

a.) $y = x^2 + 4x - 12$ vertex = $(-2, -16)$ opening direction: up

$y + 12 = x^2 + 4x$

$y + 12 + 4 = x^2 + 4x + 4$

$y + 16 = (x + 2)^2$

$y = (x + 2)^2 - 16$

b.) $y = x^2 - 9x + 8$ vertex = $(4.5, -12.25)$ opening direction: up

$y - 8 = x^2 - 9x$

$y - 8 + 20.25 = x^2 - 9x + 20.25$

$y + 12.25 = (x - 4.5)^2$

$y = (x - 4.5)^2 - 12.25$

c.) $y = x^2 + 6x + 7$ vertex = $(-3, -2)$ opening direction: up

$y - 7 = x^2 + 6x$

$y - 7 + 9 = x^2 + 6x + 9$

$y + 2 = (x + 3)^2$

$y = (x + 3)^2 - 2$

BONUS: Watch out for the signs!

d.) $y = -x^2 + 8x - 11$ vertex = $(4, 5)$ opening direction: down

$y + 11 = -x^2 + 8x$

$y + 11 = -(x^2 - 8x)$

$y + 11 - 16 = -(x^2 - 8x + 16)$

$y - 5 = -(x - 4)^2$

$y = -(x - 4)^2 + 5$

Practice:

Rewrite each standard form quadratic function into vertex form by completing the square. Then, identify the vertex and the direction of opening of the parabola.

1. $y = x^2 - 8x + 19$

$y - 19 = x^2 - 8x$

$y - 19 + 16 = x^2 - 8x + 16$

$y - 3 = (x - 4)^2$

$y = (x - 4)^2 + 3$

$V(4, 3)$

up

2. $f(x) = x^2 - 4x - 1$

$f(x) + 1 = x^2 - 4x$

$f(x) + 1 + 4 = x^2 - 4x + 4$

$f(x) + 5 = (x - 2)^2$

$f(x) = (x - 2)^2 - 5$

$V(2, -5)$

up

3. $f(x) = x^2 - 3x + 4$

$f(x) - 4 = x^2 - 3x$

$f(x) - 4 + 2.25 = x^2 - 3x + 2.25$

$f(x) - 1.75 = (x - 1.5)^2$

$f(x) = (x - 1.5)^2 + 1.75$

$V(1.5, 1.75)$

up

4. $g(x) = 2x^2 - 8x + 1$

$g(x) - 1 = 2x^2 - 8x$

$g(x) - 1 = 2(x^2 - 4x)$

$g(x) - 1 + 8 = 2(x^2 - 4x + 4)$

$g(x) + 7 = 2(x - 2)^2$

$g(x) = 2(x - 2)^2 - 7$

$V(2, -7)$

up

5. $y = x^2 + 4x - 12$

$y + 12 = x^2 + 4x$

$y + 12 + 4 = x^2 + 4x + 4$

$y + 16 = (x + 2)^2$

$y = (x + 2)^2 - 16$

$V(-2, -16)$

up

6. $h(x) = x^2 - 12x + 7$

$h(x) - 7 = x^2 - 12x$

$h(x) - 7 + 36 = x^2 - 12x + 36$

$h(x) + 29 = (x - 6)^2$

$h(x) = (x - 6)^2 - 29$

$V(6, -29)$

up

7. $y = -x^2 + 4x - 1$

$y + 1 = -(x^2 - 4x)$

$y + 1 - 4 = -(x^2 - 4x + 4)$

$y - 3 = -(x - 2)^2$

$y = -(x - 2)^2 + 3$

$V(2, 3)$

down

8. $f(x) = x^2 - 8x + 13$

$f(x) - 13 = x^2 - 8x$

$f(x) - 13 + 16 = x^2 - 8x + 16$

$f(x) + 3 = (x - 4)^2$

$f(x) = (x - 4)^2 - 3$

$V(4, -3)$

up

Homework: