Math III Similarity Review

Angle-Angle Similarity (AA) Postulate – If two angles of one triangle are congruent to two angles in another triangle, then the two triangles are similar

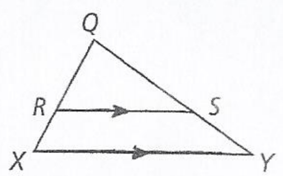
Side-Angle-Side Similarity (SAS) Postulate – If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the two triangles are similar

Side-Side-Side Similarity (SSS) Postulate – If the corresponding sides of two triangles are proportional, then the triangles are similar.

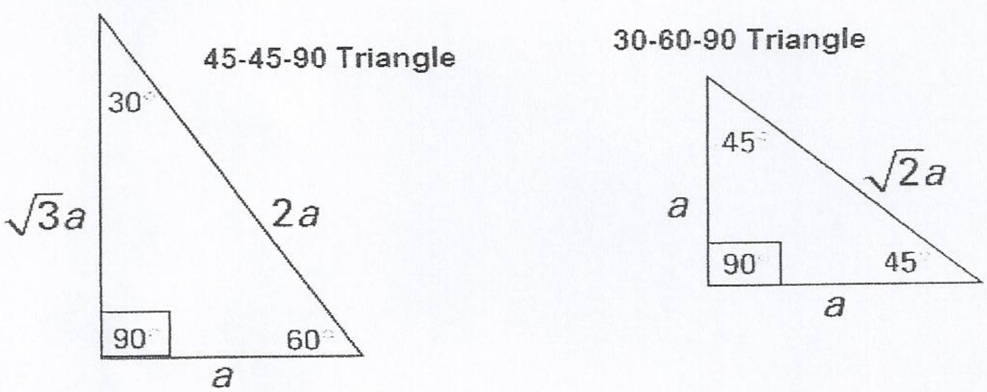
Side-Splitter Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

If $\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$, then $\dfrac{XR}{RQ} = \dfrac{SY}{SQ}$

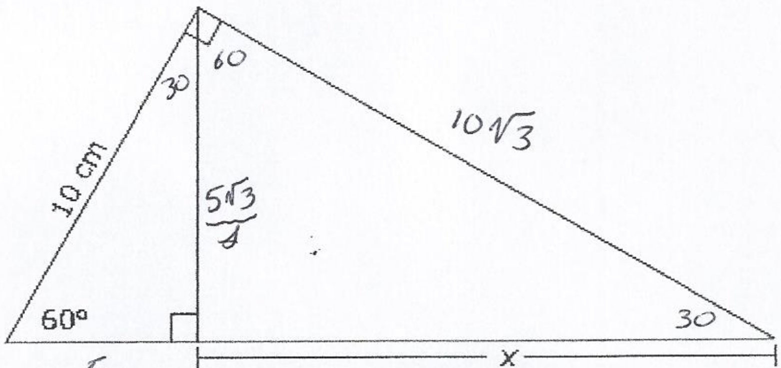


Special Right triangles



1.

What is the value of x in the triangle below?



$5\sqrt{3}\cdot\sqrt{3}$
$= 5\sqrt{9}$
$= 5\cdot 3$
$= 15$

A $\frac{5\sqrt{3}}{2}$ cm

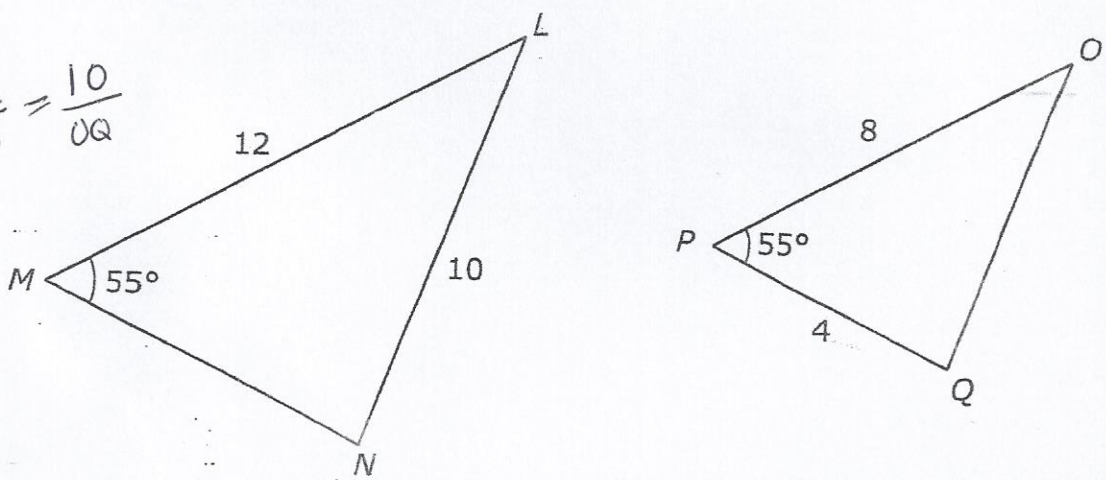
B $5\sqrt{3}$ cm

C 10 cm

D 15 cm

2. Triangles LMN and OPQ are shown below.

$\frac{12}{8} = \frac{10}{OQ}$



What additional information will prove the triangles are similar?

~~A.~~ $OQ = 6$

B. $< LMN \cong < QOP$

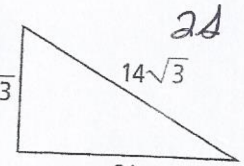
~~C.~~ $MN = 9$

D. $< NLM \cong < QOP$ proves by Similarity (AA)

3. What are the angle measures of the triangle?

(A.) 30°, 60°, and 90°   ~~C.~~ 60°, 60°, and 60°

B. 45°, 45°, and 90°   D. They cannot be determined

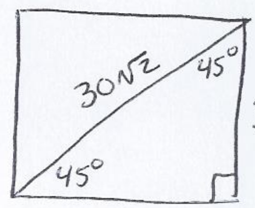


$\rightarrow 7\sqrt{3}\sqrt{3}$
$= 7\sqrt{9}$
$= 7 \cdot 3$

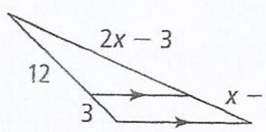
4. In the center of town there is a square park with side length 30 ft. If a person walks from one corner of the park to the opposite corner, how far does the person walk? Round your answer to the nearest foot.

A. 21 ft   (B.) 42 ft   C. 52 t   D. 60ft

$30\sqrt{2} \approx 42$



5. Solve for x



$\frac{3}{12} = \frac{x-5}{2x-3}$

$3(2x-3) = 12(x-5)$

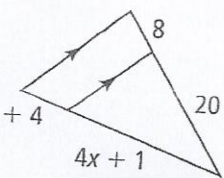
$6x - 9 = 12x - 60$

$-9 = 6x - 60$

$51 = 6x$

$x = 8.5$

6. Solve for x



$\frac{8}{20} = \frac{x+4}{4x+1}$

$8(4x+1) = 20(x+4)$

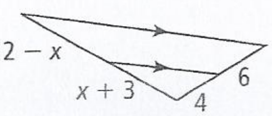
$32x + 8 = 20x + 80$

$12x + 8 = 80$

$12x = 72$

$x = 6$

7. Solve for x



$\frac{4}{6} = \frac{x+3}{12-x}$

$4(12-x) = 6(x+3)$

$48 - 4x = 6x + 18$

$+4x \quad +4x$

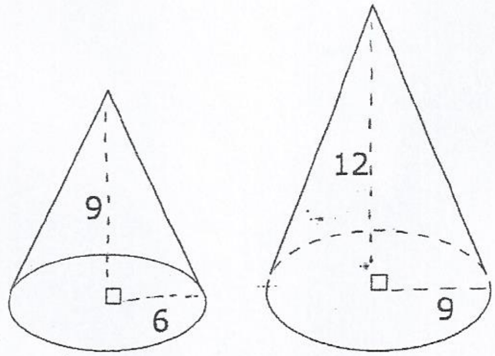
$48 = 10x + 18$

$30 = 10x$

$x = 3$

8. Which choice shows a pair of similar figures?

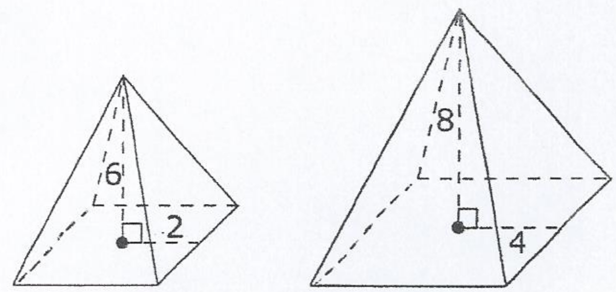
A



$\frac{9}{6} = \frac{12}{9}$

$72 = 81$ X

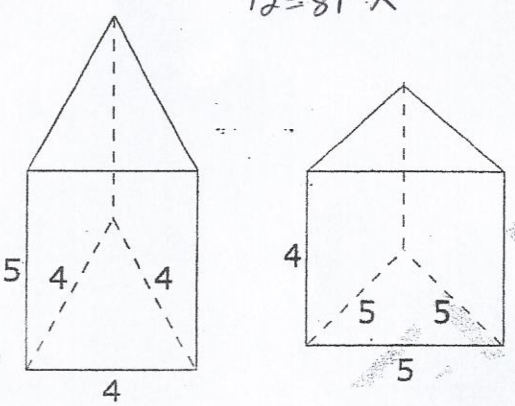
B



$\frac{6}{2} = \frac{8}{4}$

$24 = 16$ X

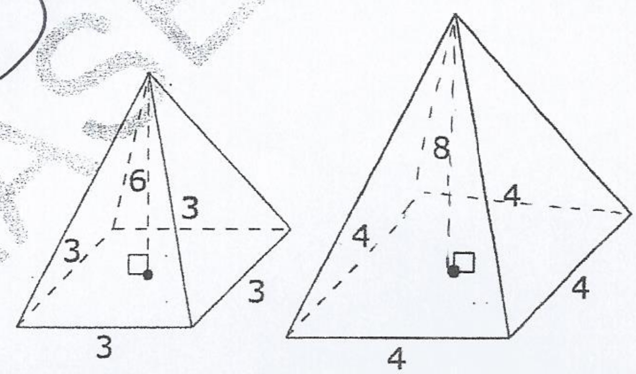
C



$\frac{4}{5} = \frac{5}{4}$

$16 = 25$ X

D

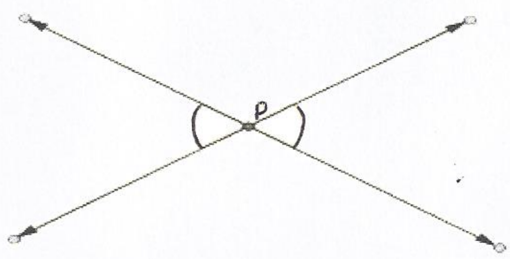


$\frac{6}{3} = \frac{8}{4}$

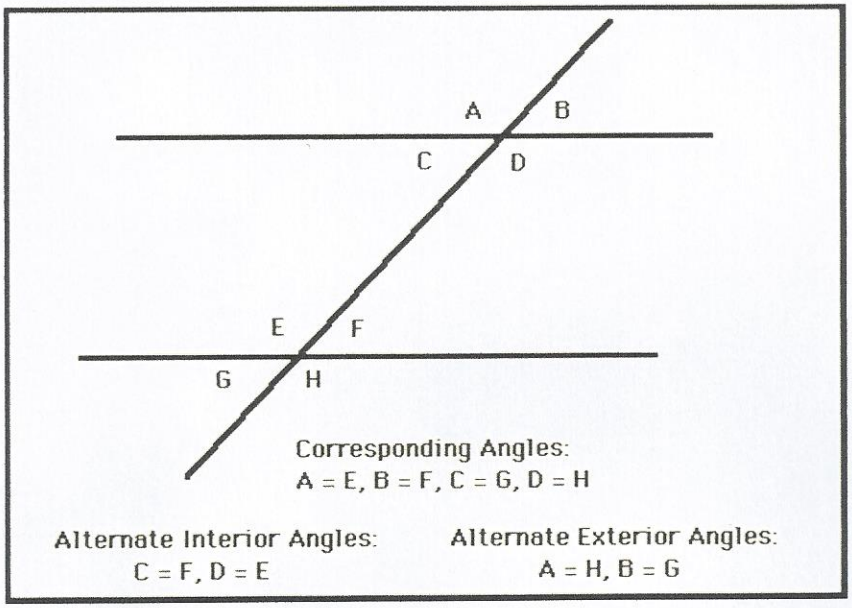
$24 = 24$ ✓

Math III Proving Geometric Theorems Review
Key

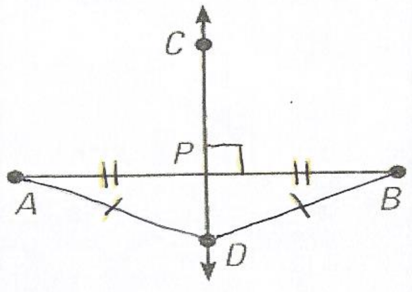
Vertical Angles – When two lines intersect, the angles across from each other are vertical angles and are congruent



Parallel lines cut by a transversal produce congruent alternate interior and corresponding angles.

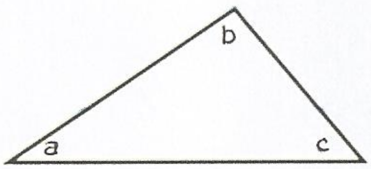


Perpendicular bisector – Points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints



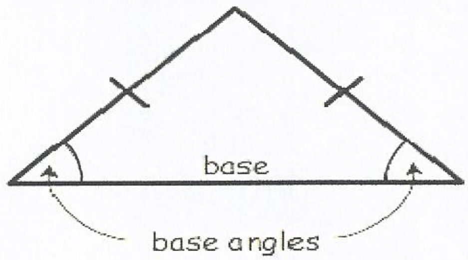
D is on $\overleftrightarrow{CP}$.

Interior angles of a triangle add up to 180^0

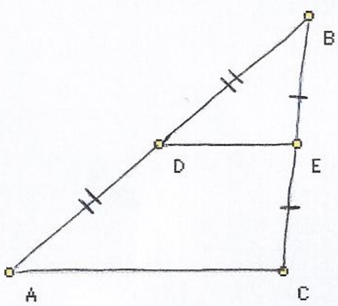


a + b + c = 180 degrees

Isosceles Triangle – base angles are congruent

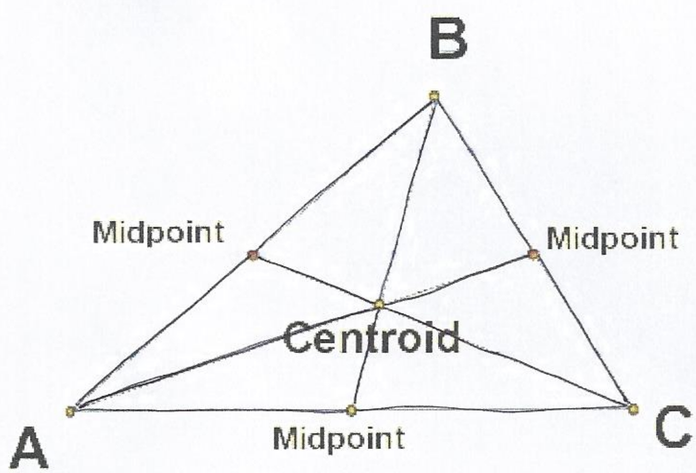


Triangle mid-segment theorem



$\overline{DE}$ is parallel to $\overline{AC}$.
$\overline{DE}$ is also half the length of $\overline{AC}$.

Centroid – where the medians of a triangle meet



The vertex to the centroid is $^2/_3$ the length of the whole segment

Parallelograms

Properties

- Opposite sides are parallel
- Opposite angles are congruent
- Opposite sides are congruent
- Diagonals bisect each other

Rectangles

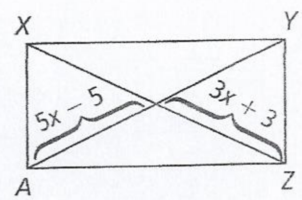
A rectangle is a type of parallelogram and it holds all the properties of a parallelogram but adds...

4 right angles

Diagonals are congruent

1. Which of the following conditions or set of conditions must be met for a parallelogram to be a rectangle?
   A. Diagonals are perpendicular
   B. Diagonals are congruent
   C. All sides are congruent
   D. The length of a diagonal is equal to the length of a side

2. For what value of x is $XYZA$ a rectangle?
   A. 2
   B. 3
   C. 4
   D. 5



$5x - 5 = 3x + 3$

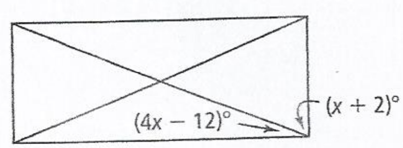
$-3x \quad -3x$

$2x - 5 = 3$

$+5 \quad +5$

$2x = 8 \quad x = 4$

3. What is the value of x for the rectangle?



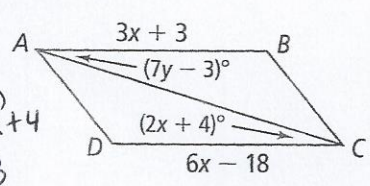
$x + 2 + 4x - 12 = 90$

$5x - 10 = 90$

$5x = 100$

$x = 20$

4. What is the value of x and y for the parallelogram?



$3x + 3 = 6x - 18$

$3 = 3x - 18$

$21 = 3x$

$x = 7$

$7y - 3 = 2x + 4$

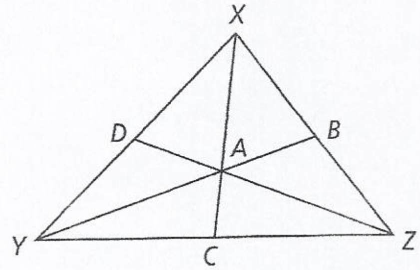
(7)

$7y - 3 = 18$

$7y = 21$

$y = 3$

5. In ΔXYZ, A is the centroid.



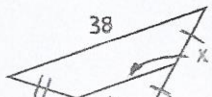
A. If DZ=12, find ZA and AD.

$ZA = 8 \quad AD = 4$

B. If ZB is 5, what other segment length can you prove? Explain how you can prove the length.

XB=5, since A is a centroid YB intersects XZ at the midpoint. So $ZB \cong XB$

6. Find the value of x

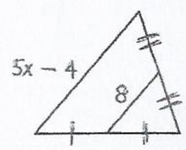


$x+2=\frac{1}{2}(38)$

$x+2=19$

$x=17$

7. Find the value of x

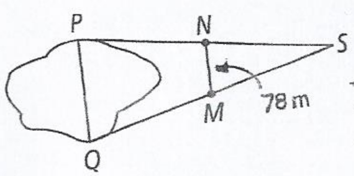


$5x-4=16$

$5x=20$

$x=4$

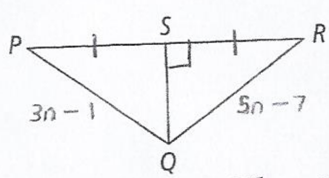
8. A surveyor needs to measure the distance PQ across the lake. Beginning at point S, she locates the midpoints, of $\overline{SQ}$ and $\overline{SP}$ at M and N. She then measures $\overline{NM}$. What is PQ?



$78(2)$

$=156$

9. What is the length of $\overline{QR}$?



$3n-1=5n-7$

$-3n \quad -3n$

$-1=2n-7$

$6=2n$

$n=3$

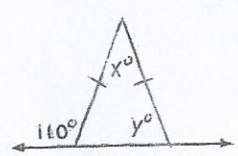
$\overline{QR}=5n-7$

$=5(3)-7$

$=15-7$

$=8$

10. Solve for x and y.

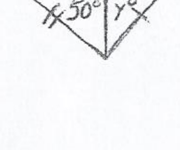


$180-110=70$

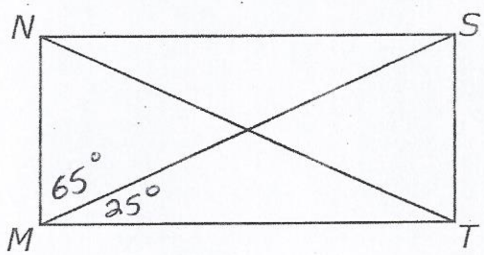
$y=70°$

$x=40°$

11. Solve for x and y.



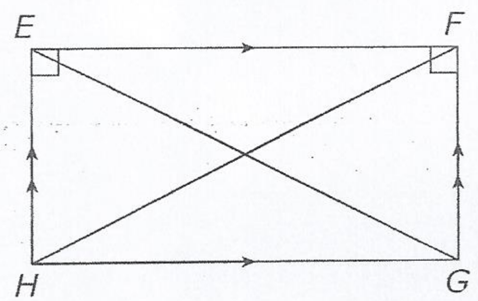
12. In the figure below, $NSTM$ is a rectangle and $m < SMN = 65$.



What is $m < NTM$?

A. 12.5 B. 25 C. 50 D. 65

13. Given:



$FG = 2x + 4$
$EG = 3x + 9$

$FH = 7x - 3$

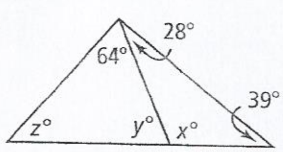
EG = FH
3x+9 = 7x−3
−3x −3x
9 = 4x−3
12 = 4x x = 3

FG = EH
= 2x+4
= 2(3)+4
= 6+4
= 10

What is the length of $\overline{EH}$?

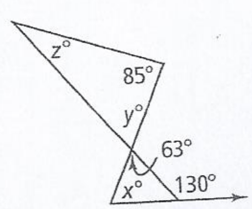
A. 18 B. 10 C. 9 D. 5

14. Find $x, y,$ and z.



X = 113
y = 67
z = 49

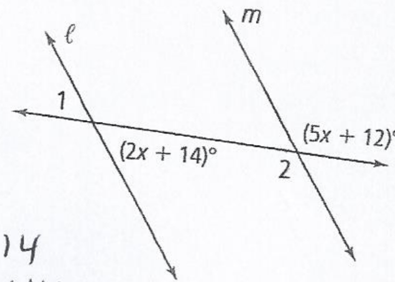
15. Find $x, y,$ and z.



X = 67
y = 63
z = 32

16. Use the figure at the right. If $l \| m$, what is the $m < 1$?

A. 22 B. 58 C. 122 D. 130



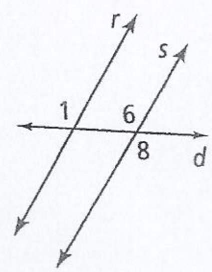
$5x+12+2x+14=180$
$7x+26=180$
$7x=154$
$x=22$

$m\angle 1 = 2x+14$
$=2(22)+14$
$=58$

17. Using the figure at the right, prove $< 1 \cong < 8$

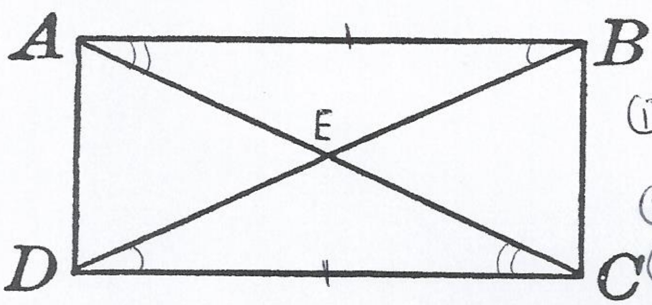
Given: $r \parallel s$

Prove: $\angle 1 \cong \angle 8$



S	R
① $r \parallel s$	① Given
② $\angle 1 \cong \angle 6$	② corresponding $\angle$'s
③ $\angle 6 \cong \angle 8$	③ vert. $\angle$'s
④ $\angle 1 \cong \angle 8$	④ Transitive property

18. Using the rectangle below, prove $\Delta ABE \cong \Delta CDE$.



S	R
① ABCD is a rectangle	① Given
② $\overline{AB} \cong \overline{CD}$	② opp. sides are $\cong$ Prop. of rectangle
③ $\overline{AB} \parallel \overline{CD}$ $\overline{AD} \parallel \overline{BC}$	③ opp. sides are $\parallel$ prop. of rectangle
④ $\angle ABD \cong \angle CDB$ $\angle BAC \cong \angle DCA$	④ alt. Interior $\angle$'s
⑤ $\Delta ABE \cong \Delta CDE$	⑤ ASA

Key

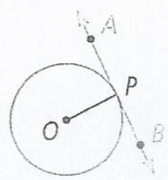
Math III Geometry Review

Circles

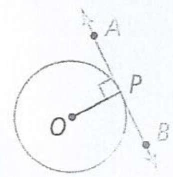
Equation of a circle $(x-h)^2 + (y-k)^2 = r^2$ where (h, k) is the center and r is the radius

A tangent line and the radius of a circle form a 90^0 angle.

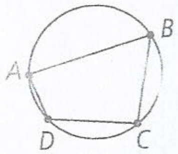
If . . .
$\overleftrightarrow{AB}$ is tangent to $\odot O$ at P



Then . . .
$\overleftrightarrow{AB} \perp \overline{OP}$

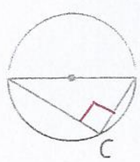


The opposite angles of a quadrilateral inscribed in a circle are supplementary

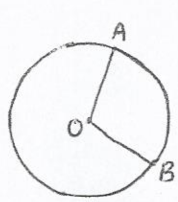


$\angle B + \angle D = 180$
$\angle A + \angle C = 180$

An angle inscribed in a semicircle is a right angle

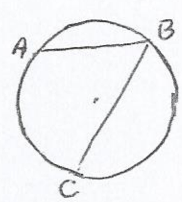


Central Angle



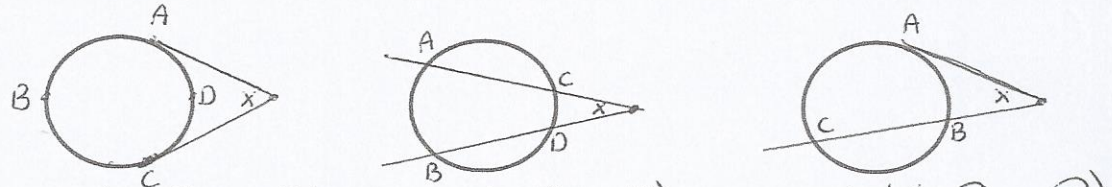
The measure of a central angle
Is equal to the arc it opens to

Inscribed Angle



The measure of the inscribed angle
is $\frac{1}{2}$ of the arc it opens to

Circumscribed Angles



$\angle x = \frac{1}{2}(m\overset{\frown}{ABC} - m\overset{\frown}{ADC})$ $\angle x = \frac{1}{2}(m\overset{\frown}{AB} - m\overset{\frown}{CD})$ $\angle x = \frac{1}{2}(m\overset{\frown}{AC} - m\overset{\frown}{AB})$

Area of a sector

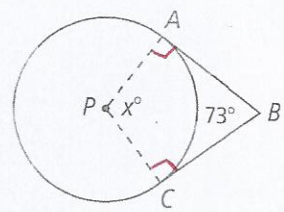
$\frac{degree}{360}\pi r^2$

Length of a sector

$\frac{degree}{360}2\pi r$

1. $\overline{AB}$ and $\overline{BC}$ are tangent to $\odot P$. What is the value of x.

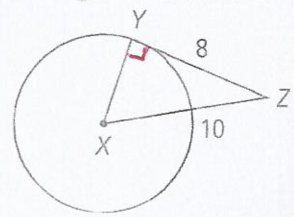
   A. 73 B. 107 C. 117 D. 146



$360-90-90-73$
$=107$

2. $\overline{YZ}$ is tangent to $\odot X$, and X is the center of the circle. What is the length of the radius of the circle?

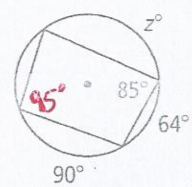
   A. 4 B. 6 C. 12 D. 12.8



$a^2+8^2=10^2$
$a^2+64=100$
$a^2=36$
$a=6$

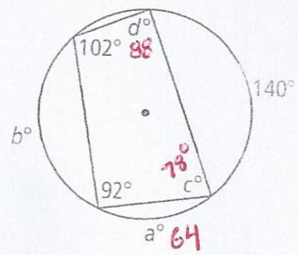
3. What is value of z?

   A. 77 B. 95 C. 126 D. 154



$95=\frac{1}{2}(z+64)$
$190=z+64$
$z=126$

4. What is the value of $a, b, c,$ and d?

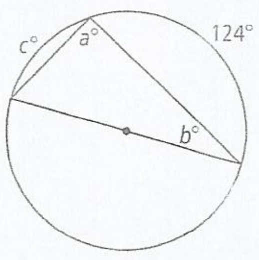


$a=64$
$b=112$
$c=78°$
$d=88°$

$102=\frac{1}{2}(140+a)$
$204=140+a$
$a=64$

$88=\frac{1}{2}(b+64)$
$176=b+64$
$b=112$

5. What is the value of $a, b,$ and c?

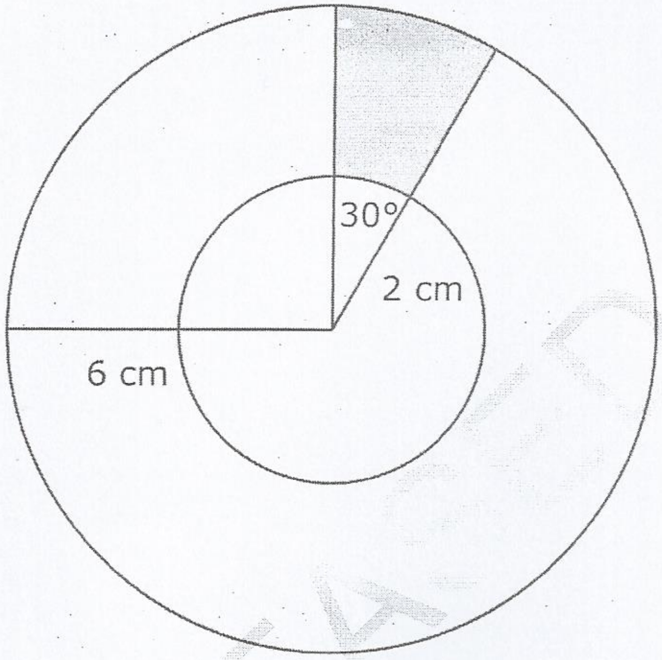


$a=90°$
$b=28°$
$c=56°$

$180-124=56$

$b=\frac{1}{2}(56)$
$b=28$

6. In the figure below, the larger circle has a radius of 6 cm, and the smaller circle has a radius of 2 cm.



$\frac{30}{360}\pi(6)^2 - \frac{30}{360}\pi(2)^2$

$= 8.377$

What is the ***approximate*** area of the shaded region?

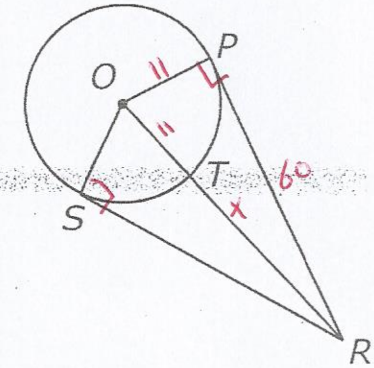
A 2.1 cm²

B 3.4 cm²

C 4.2 cm²

D 8.4 cm²

7. In the figure below, $\overline{PR}$ and $\overline{SR}$ are tangent to circle O.



$11^2+60^2=(x+11)^2$
$3721=x^2+22x+121$
$0=x^2+22x-3600$
$(x-50)(x+72)$
$x=50$ $\cancel{x=-72}$

$\overline{OR}=50+11=61$

If OT = 11 cm and PR = 60 cm, what is the length of $\overline{OR}$?

A 61 cm

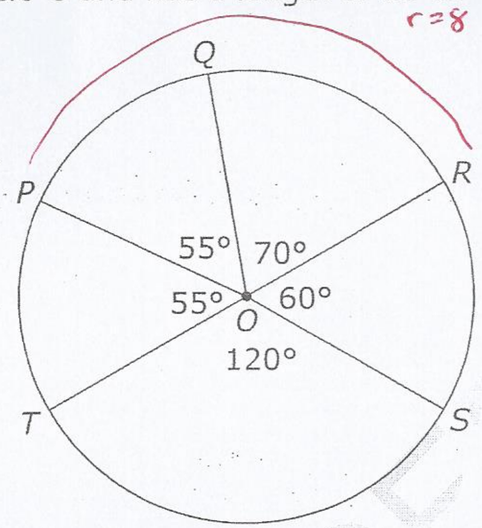
B 59 cm

C 50 cm

D 48 cm

8. $\overline{TR}$ is a diameter of circle O and has a length of 16 ft.

$r=8$



$\frac{125}{360}\pi(8^2)$

$=69.81$

What is the ***approximate*** area of the sector bounded by $\angle POR$ and $\overset{\frown}{PQR}$?

A 70 ft^2

B 67 ft^2

C 42 ft^2

D 39 ft^2

9. Derive the standard equation of the circle $x^2 + y^2 + 4x - 6y = -4$

$x^2+4x+\underline{4}+y^2-6y+\underline{9}=-4+4+9$

$\left(\frac{4}{2}\right)^2=4 \qquad \left(\frac{-6}{2}\right)=9$

$(x+2)^2+(y-3)^2=9$

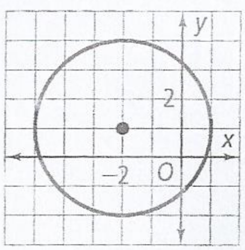
10. Which is the equation of a circle with center (-2, 3) and a radius r = 5?

A. $(x+2)^2+(y-3)^2=10$ C. $(x-2)^2+(y+3)^2=10$

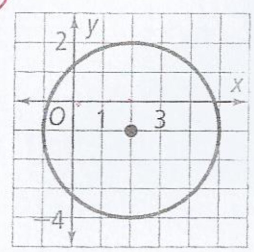
B. $(x+2)^2+(y-3)^2=25$ D. $(x-2)^2+(y+3)^2=25$

11. Which of the following is the graph of $(x-2)^2+(y+1)^2=9$ center $(2,-1)$ $r=3$

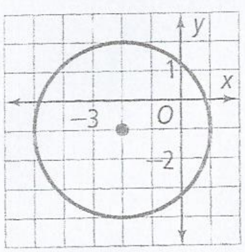
A.



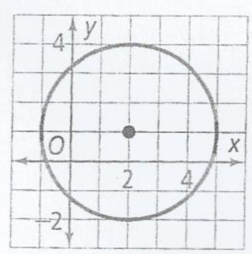
C.



B.



D.



Math III
Quadratics Review

Key

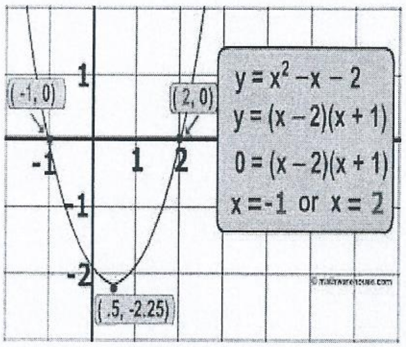
Standard form $f(x) = ax^2 + bx + c$

c is the y-intercept of a quadratic, positive a (faces up like a U), negative a (faces down)

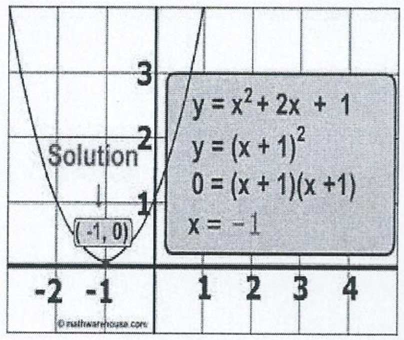
Vertex Form $f(x) = a(x-h)^2 + k \qquad vertex(h, k)$

Solutions (known as x-intercepts, zeroes, or roots) of a quadratic can be found four ways:

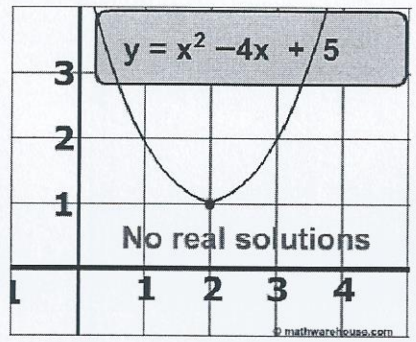
Graphing – Graph the function in y=, 2nd, trace, zero



Two Solutions



One solution



Imaginary Solutions

Factoring - transform a quadratic from standard form into factored

Ex/ Solve $f(x) = 3x^2 + 7x - 6$

Factor $f(x) = (3x-2)(x+3)$

Set each factor equal to zero and solve for x

$3x - 2 = 0 \qquad x + 3 = 0$

$3x = 2 \qquad x = -3$

$x = \frac{2}{3}$

Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, use the same *a, b, c* from standard form

Ex/ $f(x) = 3x^2 + 7x - 6 \qquad a = 3, b = 7, and\ c = -6$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(3)(-6)}}{2(3)}$$

$$= \frac{-7 \pm 11}{6}$$

So $x = \frac{2}{3}$ and $x = -3$

Completing the Square *if $a \neq 1$, divide it from the equation

Ex/ $x^2 - 6x + 25 = 0$

$x^2 - 6x \quad = -25$ *move the c value to the other side*

$x^2 - 6x + 9 = -25 + 9$ *add* $\left(\frac{b}{2}\right)^2$ *to both sides*

$(x-3)^2 = -16$ *factor the left side*

$x - 3 = \pm\sqrt{-16}$ *take the square root to both sides*

$x - 3 = \pm 4i$ *simplify the right side*

$x = 3 \pm 4i$ *solve for x*

Ex/ $3x^2 - 6x - 9 = 0$

$3x^2 - 6x \quad = 9$

$x^2 - 2x \quad = 3$

$x^2 - 2x + 1 = 3 + 1$

$(x-1)^2 = 4$

$x - 1 = \pm 2$

$x = 3, -1$

Factor by grouping

EX/ $8xy - 2x + 20y - 5$

$2x(4y-1)5(4y-1)$ split expression in half so a GCF can be pulled out and find the GCF of each side

$(4y-1)(2x+5)$ Pull out the common factor, the GCF's become a factor together

Imaginary Numbers

i is an imaginary number and occurs when a negative is under a square root

$$i^2 = -1$$

Examples

1. Simplify $\sqrt{-8}$

$i\sqrt{8}$

$4 \cdot 2$

$2 \cdot 2$

$2i\sqrt{2}$

2. Simplify $\sqrt{-32}$

$i\sqrt{32}$

$4 \cdot 8$

$2 \cdot 2 \quad 4 \cdot 2$

$2 \cdot 2$

$4i\sqrt{2}$

3. Simplify $(-1 + 5i) + (-2 - 3i)$

$-3+2i$

4. Simplify $(3 + 2i)(2 - 5i)$

$6 - 15i + 4i - 10i^2$

$+10$

$16 - 11i$

5. What are the values of x and y when $(5 + 4i) - (x + yi) = (-1 - 3i)$?

A. x = 6, y = 7   C. x = 6, y = i

B. x = 4, y = i D. x = 4, y = 7

6. Which is equivalent to $(4 - 3i)^2 + (6 + i)^2$?

A. 30   B. 50   C. $42 - 12i$ D. $62 - 12i$

$(4-3i)(4-3i) + (6+i)(6+i)$

$16 - 24i + 9i^2 \quad 36 + 12i + i^2$

$-9 \qquad -1$

$42 - 12i$

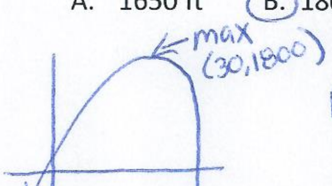
7. For which equation is the x-coordinate of the vertex at 4?

A. $f(x) = x^2 - 8x + 15$ C. $f(x) = x^2 + 6x + 8$

B. $f(x) = -x^2 - 4x + 12$ D. $f(x) = -x^2 - 2x + 2$

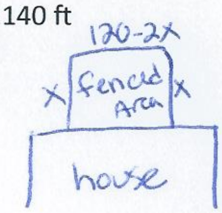
8. Adrian is using 120 feet of fencing to enclose a rectangular area for her puppy. One side of the enclosure will be her house. The function $f(x) = x(120 - 2x)$ represents the area of the enclosure. What is the greatest area that Adrian can enclose for fencing?

A. 1650 ft   B. 1800 ft   C. 1980 ft   D. 2140 ft



plug in $y = x(120 - 2x)$

find max area



If $5-3i$ is a solution then so is $5+3i$

9. If $5 - 3i$ is a solution for $x^2 + ax + b = 0$, where a and b are real numbers, what is the value of b?

A. 10 B. 14 C. 34 D. 40

$(x-(5-3i))(x-(5+3i))$

$(x-5+3i)(x-5-3i)$

$=x^2-5x-\cancel{3xi}-5x+25+\cancel{15i}+\cancel{3xi}-\cancel{15i}-\overset{+9}{\cancel{9i^2}}$

$=x^2-10x+34$

10. The graph of the function x^3 will be shifted down 2 units and to the right 3 units. Which is the function that corresponds to the resulting graph?

A. $g(x) = (x + 3)^2 + 2$ C. $g(x) = (x + 3)^2 - 2$

B. $g(x) = (x - 3)^2 + 2$ D. $g(x) = (x - 3)^2 - 2$

$y=a(x-h)^2+k$

−h right −k down

+h left +k up

11. Which choice shows the solutions to the function $8x^2 + 3x = -7$?

A. $\frac{-3\pm i\sqrt{215}}{16}$ B. $\frac{3\pm i\sqrt{215}}{16}$ C. $\frac{-3\pm\sqrt{233}}{16}$ D. $\frac{3\pm\sqrt{233}}{16}$

$\underset{a}{8x^2}+\underset{b}{3x}+\underset{c}{7}=0$

$\frac{-3\pm\sqrt{(3)^2-4(8)(7)}}{2(8)}$

$=\frac{-3\pm\sqrt{-215}}{16}=\frac{-3\pm i\sqrt{215}}{16}$

12. What value of h is needed to complete the square for the equation

$$x^2 + 10x - 8 = (x - h)^2 - 33$$

A. -25 B. -5 C. 5 D. 25

$x^2+10x-8$

$x^2+10x+25=8+25$

$(x+5)^2=33$

$(x+5)^2-33$

13. The equation $2x^2 - 5x = -12$ is rewritten in the form of $2(x - p)^2 + q = 0$. What is the value of q?

A. $\frac{167}{16}$ B. $\frac{71}{8}$ C. $\frac{25}{8}$ D. $\frac{25}{16}$

$(2x^2-5x+\_\_)+12=0$

$2(x^2-\frac{5}{2}x+\frac{25}{16})+12-\frac{25}{8}$

$(\frac{96}{8})$

$2(x-\frac{5}{4})^2+\frac{71}{8}=0$

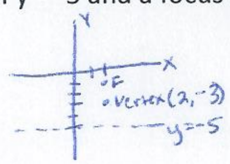
*The vertex is halfway between the focus and directrix

* y value of focus = y value of vertex $+\frac{1}{4a}$

14. Which is an equation of a parabola what has a directrix of y = -5 and a focus at (2, -1)?

A. $y = \frac{1}{2}(x + 2)^3 + 2$ C. $y = \frac{1}{8}(x + 2)^3 + 3$

B. $y = \frac{1}{8}(x - 2)^3 - 3$ D. $y = \frac{1}{2}(x - 2)^3 - 2$

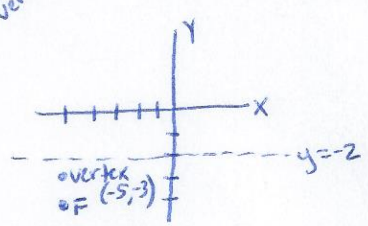


$-1=-3+\frac{1}{4a}$

$\frac{2}{1}=\frac{1}{4a}$

$\frac{1=8a}{8\quad 8}$ $a=1/8$

15. Find the equation of a parabola with a focus of (-5, -4) and a directrix of y = -2



Vertex $(-5,-3)$

$y=-\frac{1}{4}(x+5)^2-3$

$-4=-3+\frac{1}{4a}$

$+3\quad +3$

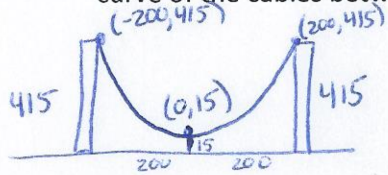
$\frac{-1}{1}=\frac{1}{4a}$

$1=-4a$

$\overline{-4}\quad\overline{-4}$

$a=-\frac{1}{4}$

16. A suspended bridge has two cables secured at either end of the span by two supporting towers. The cables are attached to the tops of the towers. In the section between the two towers, the cables form a parabolic curve. At their lowest point, the cables are 15 feet from the surface of the bridge. The towers are 400 feet apart, and the vertical distance from the surface of the bridge to the top of each tower is 415 feet. What is the quadratic equation that describes the curve of the cables between the towers? Use the lowest point as the y-intercept.



Can use QuadReg or by hand

$\boxed{\text{Stat}}$ $\boxed{\text{edit}}$ $\boxed{\text{Stat}}$

$\boxed{\text{calc}}$ $\boxed{\text{quadreg}}$

$a = .01$
$b = 0$
$c = 15$

$y = .01x^2 + 15$

or

$y = \frac{1}{100}x^2 + 15$

$y = a(x-h)^2 + k$

$415 = a(200-0)^2 + 15$

$415 = 40{,}000a + 15$

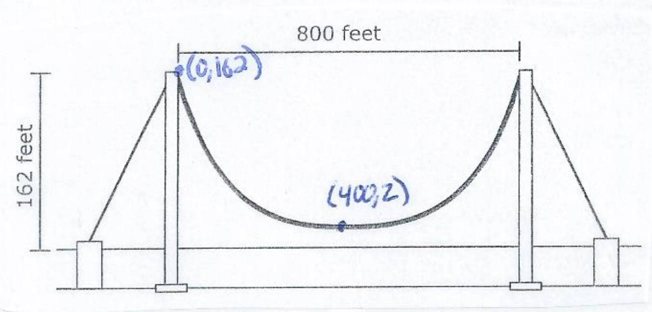
$\frac{400}{40{,}000} = \frac{40{,}000a}{40{,}000}$

$a = \frac{400}{40{,}000} = \frac{1}{100}$

$y = \frac{1}{100}x^2 + 15$

17. The towers of a suspended bridge are 800 feet apart and rise 162 feet higher than the road. Suppose that the cable between the towers has the shape of a parabola and is 2 feet higher than the road at the point halfway between the towers.

can use y axis on a tower or the lowest point. I used the first tower as y.



$y = a(x-h)^2 + k$

$162 = a(0-400)^2 + 2$

$162 = 160{,}000a + 2$

$\frac{160}{160{,}000} = \frac{160{,}000a}{160{,}000}$

$a = \frac{1}{1000}$

$y = \frac{1}{1000}(x-400)^2 + 2$

plug in $x = 120$

$y = \frac{1}{1000}(120-400)^2 + 2$

$y = 80.4$

← Since my left tower is at $x=0$ then 120' from the tower is $x = 120$

What is the approximate height of the cable 120 feet from either tower?

A. 80 feet B. 74 feet C. 22 feet D. 16 feet

18. A square and a rectangle have the same area. The length of the rectangle is five inches more than twice the length of the side of the square. The width of the rectangle is 6 inches less than the side of the square. Find the length of the side of the square.

x $2x+5$ $x-6$

$\frac{\text{Area}}{L \cdot w} = \frac{\text{Area}}{L \cdot w}$

$x^2 = (x-6)(2x+5)$

$x^2 = 2x^2 - 7x - 30$

$-x^2 \quad -x^2$

$0 = x^2 - 7x - 30$

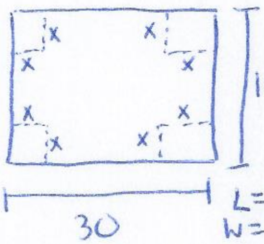
$(x+3)(x-10)$

$\cancel{x=-3}$ $\underline{x=10}$

can't happen

19. A cardboard box company has been contracted to manufacture open-top rectangular storage boxes for a manufacturing company. The company has 30 cm X 16 cm cardboard sheets. They plan to cut a square from each corner of the sheet and bend up the sides to form the box. If the company wants to make boxes with the largest possible volume: $y=(30-2x)(16-2x)(x)$ graph

max (3.33, 726)



$L=30-2x$
$W=16-2x$
$H=x$

- Find the dimensions of the square being cut out.
  $x=3.33$ 3.33   3.33 by 3.33
- What are the dimensions of the box? $L=30-2x$ $=30-2(3.33)$ $=23.34$ cm $W=16-2x$ $=16-2(3.33)$ $=9.34$ cm $H=3.33$ cm
- What is the maximum volume of the box?
  $726 cm^3$

20. A box with an open top will be constructed from a rectangular piece of cardboard.
    - The piece of cardboard is 8 inches by wide and 12 inches long.
    - The box will be constructed by cutting out equal squares of sides x at each corner and then folded up at the sides.

$L=12-2x$
$W=8-2x$
$H=x$

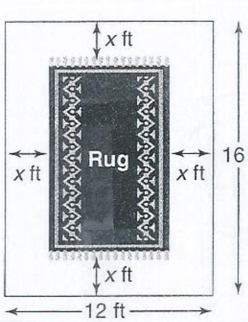
$y=(12-2x)(8-2x)(x)$ graph

What is the entire domain for the function $V(x)$ that gives the volume of the box as a function of x?

A. $0 < x < 4$ B. $0 < x < 6$ C. $0 < x < 8$ D. $0 < x < 12$

4

21. Abey Numkena is an interior designer. She has been asked to locate an oriental rug for a new corporate office. As a rule, the rug should cover $\frac{1}{2}$ of the total floor area with a uniform width surrounding the rug. $16 \cdot 12 = 192$ rug $= \frac{1}{2}(192) = 96$



- If the dimensions of the room are 12 feet by 16 feet, write an equation to model the situation. $y=(12-2x)(16-2x)-96$ or $y=4x^2-56x+96$
- Sketch a graph of the function.
  (2,0)   (12,0) extraneous $x=2$
- What are the dimensions of the rug?
  $L=(16-2x)$ $=16-2(2)$ $=12$ $W=(12-2x)$ $=12-2(2)$ $=8$ 8×12

Key

Math III Review
Polynomials A-APR.1

Adding polynomials – Add like terms, the exponents don't change!

Ex/ $(3x^2 - 4 + 2x) + (5x - 6x^2 + 7) = -3x^2 + 7x + 3$

Subtracting polynomials – Keep the first polynomial the same, change the subtraction to addition, and change the signs of the second polynomial. Exponents don't change.

Ex/ $(3x^2 - 4 + 2x) - (5x - 6x^2 + 7) = 9x^2 - 3x - 11$

Multiplying Polynomials – Each term in a polynomial has to be multiplied to each term in the other polynomial. Exponents change when terms are multiplied.

Ex/ $(2x^2 - 6x + 1)(x + 3)$

$2x^3 + 6x^2 - 6x^2 - 18x + x + 3$

$2x^3 - 17x + 3$

Ex/ $(x + 5)(x - 2)(3x + 4)$

$x^2 - 2x + 5x - 10\ (3x + 4)$

$(x^2 + 3x - 10)(3x + 4)$

$3x^3 + 4x^2 + 9x^2 + 12x - 30x - 40$

$3x^3 + 13x^2 - 18x - 40$

Dividing Polynomials – Can divide using synthetic or long division

Synthetic

$(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$

$$\begin{array}{r|rrrr} 4 & 2 & -13 & 26 & -24 \\ & & 8 & -20 & 24 \\ \hline & 2 & -5 & 6 & 0 \end{array}$$

Long Division

$(2x^3 - 13x^2 + 26x - 24) \div (x - 4)$

$$\begin{array}{r} 2x^2 - 5x + 6 \\ x - 4\,\overline{)\,2x^3 - 13x^2 + 26x - 24} \\ \underline{(-)\,2x^3 - 8x^2} \qquad\qquad \\ -5x^2 + 26x \qquad \\ \underline{(-)\,-5x^2 + 20x} \qquad \\ 6x - 24 \\ \underline{(-)\,6x - 24} \\ 0 \end{array}$$

Roots, Zeroes, X-Intercepts – Are all solutions to polynomials

Finding the polynomial given the roots

Ex/ Find a 3rd degree polynomial given the roots 2 and $3i$

$$(x-2)(x-3i)(x+3i)$$

$$(x-2)(x^2+3xi-3xi-9i^2)$$

$$(x-2)(x^2+9)$$

$$x^3-2x^2+9x-18$$

Ex/ Find the roots of $x^3-4x^2+4x-16$

-Graph the polynomial and find a zero. This polynomial crosses the x-axis at 4.

-Divide $(x-4)$ from the polynomial.

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 4 & -16 \\ & & 4 & 0 & 16 \\ \hline & 1 & 0 & 4 & 0 \end{array}$$

-So the polynomial is reduced to x^2+4. Can use either QF or solve the square root equation.

$$x^2+4=0$$

$$-4 \quad -4$$

$$x^2=-4$$

$$\sqrt{x^2}=\pm\sqrt{-4}$$

$$x=\pm 2i$$

So the roots of the polynomial are 4, $2i$, $-2i$

Review Examples

1. Which expression is equivalent to $(x+3)^3-9x(x+3)$?
   A. x^3+27
   B. x^3-27
   C. $x^3-9x^2-27x+27$
   D. $x^3-9x^2+27x+27$

$(x+3)(x+3)(x+3)-9x(x+3)$

$(x^2+6x+9)(x+3)-9x(x+3)$

$x^3+3x^2+6x^2+18x+9x+27$

$(x^3+\cancel{9x^2}+\cancel{27x}+27)-(\cancel{9x^2}+\cancel{27x})$

x^3+27

2. The volume of a rectangular prism is represented by the expression $(x^3 - 2x^2 - 20x - 24)$. If the length is $(x - 6)$ and the height and width are equal, what is the width of the prism?

A. $x + 2$
B. $x - 2$
C. $x + 4$
D. $x - 4$

Since $V = L \cdot W \cdot H$ divide out the length

6| 1 -2 -20 -24
6 24 24
1 4 4 0

x^2+4x+4

$(x+2)(x+2)$ ← one is height the other is width

So $H \cdot W = x^2+4x+4$

3. Suppose $p(x) = x^3 - 2x^2 + 13x + k$. The remainder of the division of $p(x)$ by $(x + 1)$ is -8. What is the remainder of the division of $p(x)$ by $(x - 1)$?

A. -8
B. 8
C. 16
D. 20

-1| 1 -2 13 K
-1 3 -16
1 -3 16 -8

So K = 8

So now divide out (x-1) from $x^3-2x^2+13x+8$

1| 1 -2 13 8
1 -1 12
1 -1 12 20

↑ Remainder

4. Which expression is the factored form of $x^3 + 2x^2 - 5x - 6$?

A. $(x + 1)(x + 1)(x - 6)$
B. $(x + 2)(2x - 5)(x - 6)$
C. $(x + 3)(x + 1)(x - 2)$
D. $(x - 3)(x - 1)(x + 2)$

$(x+3)(x+1)(x-2)$

$(x^2+4x+3)(x-2)$

$x^3-2x^2+4x^2-8x+3x-6$

$=x^3+2x^2-5x-6$

5. What are the zeroes of the polynomial function $y = 2x^3 - 7x^2 + 2x + 3$?

A. $\frac{1}{2}, 1, 3$ B. -1, 1, 3 C. $-\frac{1}{2}, 1, 3$ D. -3, $\frac{1}{2}$, 1

↑ graph in calc and look at where it crosses the x-axis

6. Which polynomial function has zeroes at -4, 3, and 5?

A. $f(x) = (x + 4)(x + 3)(x + 5)$
B. $g(x) = (x + 4)(x - 3)(x - 5)$
C. $h(x) = (x - 4)(x - 3)(x - 5)$
D. $k(x) = (x - 4)(x + 3)(x + 5)$

7. Which is not a factor of $x^3 - x^2 - 17x - 15$?

A. $x - 5$ B. $x + 1$ C. $x + 3$ D. $x + 5$

← graph in calc and it crosses the x-axis at -3, -1, 5 so the factors are $(x+3)(x+1)(x-5)$

8. Which of the following is not a solution of $x^4 - 3x^2 - 54 = 0$?

A. -3 B. 3 C. $-3i$ D. $-i\sqrt{6}$

← graph in calc it crosses the x-axis at -3 & 3. Since those are roots divide them out of the equation

-3| 1 0 -3 0 -54
-3 9 -18 54
1 -3 6 -18 0

$x^3-3x^2+6x-18$

3| 1 -3 6 -18
3 0 18
1 0 6 0

$x^2+6=0$

Solve

$x^2=-6$

$x=\pm\sqrt{-6}$

$x=\pm i\sqrt{6}$

$(a-b)(a-b)(a-b)$

$(a^2-2ab+b^2)(a-b)$

$a^3-a^2b-2a^2b+2ab^2+ab^2-b^3$

$=a^3-3a^2b+3ab^2-b^3$

9. What is the expanded form of $(a-b)^3$?
   A. $a^3+a^2b+ab^2+b^3$
   B. $a^3+3a^2b+3ab^2+b^3$
   C. $a^3-a^2b+ab^2-b^3$
   D. $a^3-3a^2b+3ab^2-b^3$

10. The function f is defined as $f(x)=6x^4+x^3+4x^2+x-2$.
   - Using the Remainder Theorem, determine if $\frac{1}{2}$ is a root of $f(x)$. Explain.

     Synthetic division. If ½ is a root then the remainder when it is divided out is zero

     $$\begin{array}{r|rrrrr} \frac{1}{2} & 6 & 1 & 4 & 1 & -2 \\ & & 3 & 2 & 3 & 2 \\ \hline & 6 & 4 & 6 & 4 & 0 \end{array}$$

     ← yes it is a root

   - If i is also a root, what are the other two roots?

     imaginary root have a conjugate pair so another root is $-i$. The other root is a real number and when the function is graphed it shows it as $x=-2/3$

     So the 4 roots are: $1/2, -2/3, i, -i$

11. For a certain polynomial function, x = 3 is a zero with multiplicity of two and x = -3 is a zero with a multiplicity of one. Write a possible equation for this function and sketch its graph.

    multiplicity means two roots that are the same. So this function has two roots of 3.

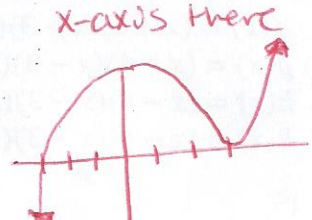
    roots = 3,3,-3 so it crosses the x-axis there

    $(x-3)(x-3)(x+3)$

    $(x^2-6x+9)(x+3)$

    $x^3+3x^2-6x^2-18x+9x+27$

    $x^3-3x^2-9x+27$



12. Is $(2x-3)^3-64$ equivalent to $(2x-11)(2x+5)$? Explain your reasoning.

    $(2x-3)(2x-3)(2x-3)-64$

    $(4x^2-12x+9)(2x-3)-64$

    $8x^3-12x^2-24x^2+36x+18x-27-64$

    $=8x^3-36x^2+54x-91$

    $=4x^2+10x-22x-55$

    $=4x^2-12x-55$

    no

Key

Math III Review Rational and Radical Equations

Simplifying Rational and Radical Equations

Ex/ Solve $2x = \sqrt{5x-1} + 1$

Step 1: Subtract 1 from each side to isolate the radical term

$$2x - 1 = \sqrt{5x-1}$$

Step 2: Square both sides

$$4x^2 - 4x + 1 = 5x - 1$$

Step 3: Set the right side equal to zero

$$4x^2 - 9x + 2 = 0$$

Step 4: Solve for x (factoring, quadratic formula, graphing)

$$x = \frac{1}{4} \quad and \quad x = 2$$

Step 5: Plug answers back into the original equation and check for extraneous solutions

$2\left(\frac{1}{4}\right) = \sqrt{5\left(\frac{1}{4}\right) - 1} + 1$

$\frac{1}{2} \neq 1\frac{1}{2}$

So $x = \frac{1}{4}$ is **not** a solution

$2(2) = \sqrt{5(2) - 1} + 1$

$4 = 4$

So $x = 2$ is a solution

The solution $\frac{1}{4}$ is an **extraneous solution** because is a solution to the transformed equation, not to the original equation

Ex/ Solve $\frac{x}{x-1} - 1 = \frac{x}{2}$

Step 1: Get a common denominator, in this case $2(x-1)$

$$\frac{2x}{2(x-1)} - \frac{2(x-1)}{2(x-1)} = \frac{x(x-1)}{2(x-1)}$$

Step 2: Since the denominators are the same we only need to simplify the numerator

$$2x - 2(x-1) = x(x-1)$$

$$2x - 2x + 2 = x^2 - x$$

$$0 = x^2 - x - 2$$

Step 3: Solve for x

$$0 = (x-2)(x+1)$$

So *x* = 2 *and x* = -1

Step 4: Plug answers back into the original equation and check for extraneous solutions

$$\frac{2}{2-1} - 1 = \frac{2}{2} \qquad \frac{-1}{-1-1} - 1 = \frac{-1}{1}$$

$$1 = 1 \qquad -\frac{1}{2} \neq -1$$

So x = 2 is a solution   So x = -1 is **not** a solution

The solution -1 is an **extraneous solution**

Examples:

1. Solve for x: $\left(\frac{x+1}{5} - 2 = \frac{-4}{x}\right)5x$
   A. x = 4
   B. x = 5
   C. x = 4, 5
   D. no solution

$x^2 + x - 10x = -20$

$+20 \quad +20$

$x^2 - 9x + 20 = 0$

$(x-5)(x-4) = 0$

$x = 5, 4$

when they ere plugld in both work

2. Solve for x: $\left(\frac{8}{x-5} - \frac{9}{x-4} = \frac{5}{x^2-9x+20}\right)(x-5)(x-4)$

$(x-5)(x-4)$

$(8x-32)-(9x-45)=5$

$-x+13=5$

$-13 \quad -13$

$-x=-8$

$x=8$

when pluged in $x=8$ works

3. Solve for x: $8-\sqrt{x+12}=3$

$-8 \qquad -8$

$\frac{-\sqrt{x+12}}{-1} = \frac{-5}{-1}$

$\sqrt{x+12}^2 = 5^2$

$x+12=25$

$-12 \quad -12$

$x=13$ when pluged in

$x=13$ works

4. Solve for x: $(\sqrt{x+15})^2=(5+\sqrt{x})^2$

$x+15=25+10\sqrt{x}+x$

$-x \qquad -x$

$15=25+10\sqrt{x}$

$-25 \quad -25$

$\frac{-10}{10}=\frac{10\sqrt{x}}{10}$

$\sqrt{x}=-1$

NS

Key

Graphing Rational Functions

A **rational function** are functions where $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ *and* $b(x)$ are both polynomials and $b(x) \neq 0$.

$f(x)$ has a **vertical asymptote** whenever $b(x) = 0$

$f(x)$ has at most one **horizontal asymptote**

- If the degree of the numerator is greater than the denominator, there is no horizontal asymptote.
- If the degree of the numerator is less than the denominator, the horizontal asymptote is the x-axis (y = 0)
- If the degree of the numerator equals the denominator, the horizontal asymptote is the line $y = \frac{leading\ coefficint\ of\ numerator}{leading\ coefficient\ of\ denominator}$

Points of discontinuity (hole) – Occurs when the numerator and denominator have a common factor.

Examples:

1. Describe the asymptotes and points of discontinuity of $f(x) = \frac{x^2+4x-5}{x+5}$ $= \frac{(x+5)(x-1)}{x+5}$ hole

point of discontinuity (hole) $x=-5$

no horizontal asymptote

no vertical asymptote since the $(x+5)$'s cancel the result is $f(x)=x-1$ which is a line.

2. Given the function: $g(x) = \frac{(x-2)(3x+2)}{(x+4)(x-2)(x-6)}$

→ numerator x^2 denominator x^3 so horizontal asymptote $y=0$ (x-axis)

vertical asymptotes $x=-4, x=6$

- What are the equations of the asymptotes of this function?
- Determine if there are any points of discontinuity. Explain why or why not. yes @ $x=2$ since the function has an $x-2$ in the numerator and denominator
- Describe the end behavior as x approaches - ∞ and as x approaches + ∞

$x \to -\infty \quad g(x) \to -\infty$

$x \to +\infty \quad g(x) \to +\infty$

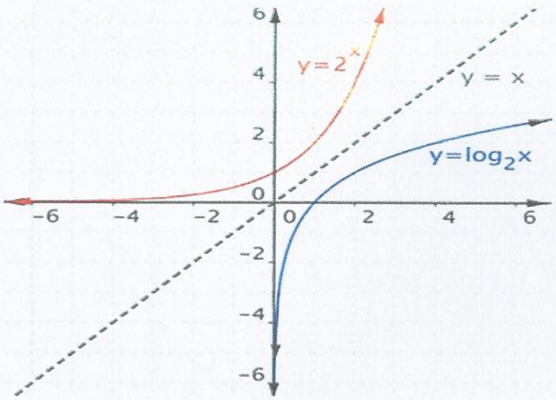
Key

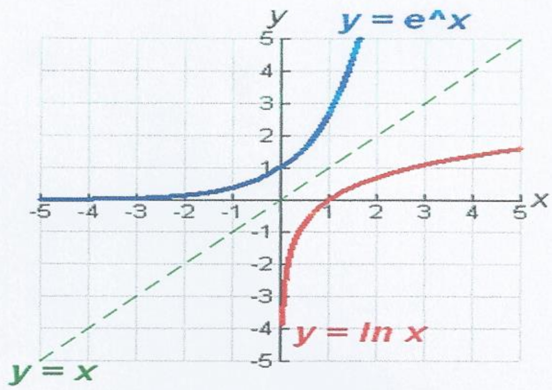
Math III Logs and Exponential

Transforming from exponential form to logarithmic form

Exponential Form	Logarithmic Form
$y = b^x$	$y = \log_b x$
$y = e^x$	$y = \ln x$

$y = b^x$ and $y = \log_b x$ are inverses of each other as well as $y = e^x$ and $y = \ln x$. Their graphs are reflections across the $y = x$ line





Logarithm Rules

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^y) = y\log_b x$

$\log_b x = \frac{\log x}{\log b}$

1. Which is the function $2^{x-1} = 8$ written in logarithmic form?
   A. $\log_2 x - 1 = 8$ C. $\log_8 x - 1 = 2$
   B. $x = \log_2 8 + 1$ D. $x = \log_2 8 - 1$

$2^{x-1} = 8$

$\log_2 8 = x - 1$

$+1 \quad +1$

$x = \log_2 8 + 1$

$$\frac{4500}{2500} = \frac{2500e^{.08x}}{2500}$$

$$\frac{9}{5} = e^{.08x} \rightarrow \frac{\ln\left(\frac{9}{5}\right)}{.08} = \frac{.08x}{.08}$$

$$x = \frac{\ln\left(\frac{9}{5}\right)}{.08}$$

2. Sally opened a savings account that earns 8% interest compounded continuously in order to save money for a \$4500 car. So far Sally has saved \$2500. How many years did it take for Sally to save enough money to buy the car if she did not add any more money to the account?

A. $x = \frac{\ln\left(\frac{9}{5}\right)}{.08}$ B. $x = \frac{.08}{\ln\left(\frac{9}{5}\right)}$ C. $x = \log_{1.08}\left(\frac{9}{5}\right)$ D. $x = \log_{\frac{9}{5}} 1.08$

3. Which of the following is equivalent to $e^{4x} = 2981$?

A. $x = \frac{\ln 2981}{4}$ B. $x = \frac{4}{\ln 2981}$ C. $x = \frac{\ln 4}{2981}$ D. $x = \frac{2981}{\ln 4}$

$$\ln e^{4x} = \ln 2981$$

$$\frac{4x}{4} = \frac{\ln 2981}{4}$$

$$x = \frac{\ln 2981}{4}$$

4. Which of the following is equivalent to $2^{3x-4} = 32$?

A. $x = \frac{\log_2 32}{3} + 4$ B. $x = \frac{\log_2 32+4}{3}$ C. $\log_2 3x - 4 = 32$

$$2^{3x-4} = 32$$

$$\log_2(32) = 3x - 4$$

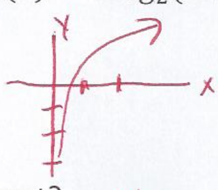
$$+4 \qquad +4$$

$$\frac{\log_2(32)+4}{3} = \frac{3x}{3}$$

$$x = \frac{\log_2(32)+4}{3}$$

5. Given the function: $f(x) = 2\log_2(2x)$

A. Sketch the graph



B. State the x-intercept? $x = 1/2$

C. State the domain and range? Domain $x > 0$

Range $\mathbb{R}$

D. Describe the end behavior as x approaches ∞.

$x \rightarrow +\infty \quad f(x) \rightarrow +\infty$

Key

Math III Trigonometry Review

F.TF.1, 2, 5, & 8

Trig Ratios

$$Sin\theta = \frac{Opposite}{Hypotenuse} \qquad Csc\theta = \frac{Hypotenuse}{Opposite}$$

$$Cos\theta = \frac{Adjacent}{Hypotenuse} \qquad Sec\theta = \frac{Hypotenuse}{Adjacent}$$

$$Tan\theta = \frac{Opposite}{Adjacent} \qquad Cot\theta = \frac{Adjacent}{Opposite}$$

Trig Identities

$$Tan\theta = \frac{sin\theta}{cos\theta} \qquad Cot\theta = \frac{cos\theta}{sin\theta}$$

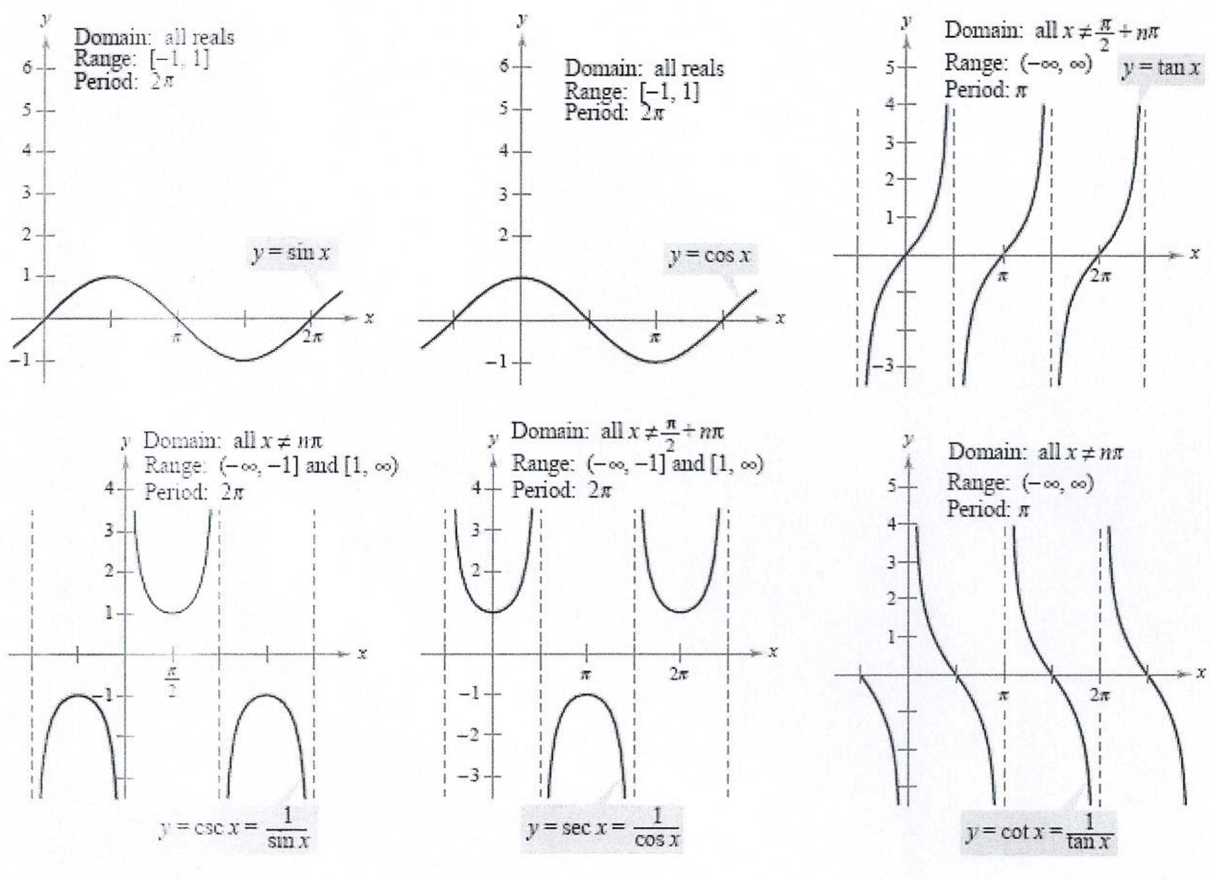
$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

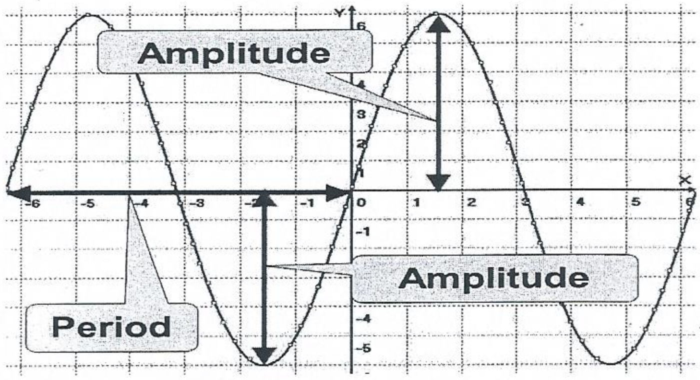
$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Trig Graphs



The graphs of the six trigonometric functions



Amplitude - half of the distance from the maximum and minimum

Period – The horizontal length of one complete cycle

Frequency – The number of cycles the function completes in a given interval.

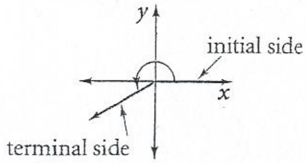
Midline – The horizontal line half way between the maximum and minimum.

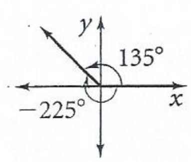
Graph Trigonometric Functions

$$y = \overset{\textbf{amplitude}}{\overset{\downarrow}{a}} \sin \overset{\textbf{period}}{\overset{\downarrow}{b}}(\theta - \underset{\textbf{phase shift}}{\underset{\uparrow}{h}}) + \underset{\textbf{vertical shift}}{\underset{\uparrow}{k}}$$

Coterminal angles – two angles in standard position that have the same terminal side.

Standard Position





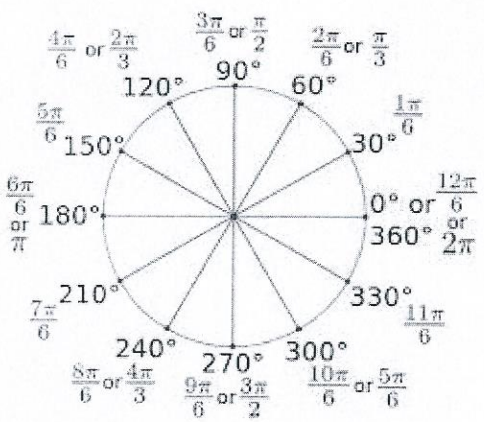
Angles that have measures 135° and −225° are coterminal.

1 radian $\approx$ 57^0

To change from degrees to radians – multiply the degrees by $\frac{\pi}{180}$

To change from radians to degrees – multiply the radian by $\frac{180}{\pi}$

Unit Circle



Review Problems

1. Which expression is equivalent to $\sin\theta \cos\theta \csc\theta$?
   A. $\sin\theta$ B. $\cos\theta$ C. $\sec\theta$ D. $\tan\theta$

$= \cancel{\sin\theta}\cos\theta \frac{1}{\cancel{\sin\theta}} = \cos\theta$

2. Which expression is equivalent to $\cos\theta + \tan\theta \sin\theta$?
   A. $\sec\theta$ B. $\tan\theta$ C. $\sin\theta$ D. $\cos\theta$

$= \cos\theta + \frac{\sin\theta}{\cos\theta} \cdot \sin\theta = \frac{\cos\theta}{1} \cdot \cos\theta + \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} = \sec\theta$

3. Which expression is equivalent to $\frac{\cos\theta}{1-\sin\theta} - \tan\theta$?
   A. $\sec\theta$ B. $\sin\theta$ C. $\cos\theta$ D. $\csc\theta$

$\frac{\cos\theta}{1-\sin\theta} - \tan\theta$

$\frac{\cos\theta}{1-\sin\theta} \cdot \cos\theta - \frac{\sin\theta}{\cos\theta}(1-\sin\theta)$

$\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)} - \frac{\sin\theta - \sin^2\theta}{\cos\theta(1-\sin\theta)}$

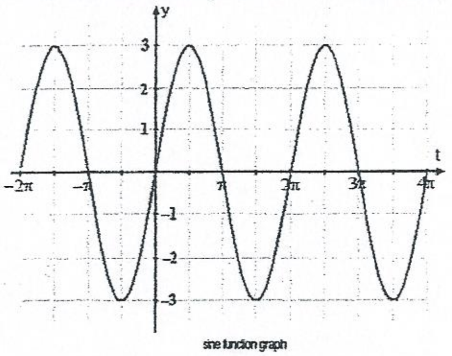
$\frac{\underbrace{\cos^2\theta + \sin^2\theta}_{1} - \sin\theta}{\cos\theta(1-\sin\theta)} = \frac{\cancel{1-\sin\theta}}{\cos\theta\cancel{(1-\sin\theta)}} = \frac{1}{\cos\theta} = \sec\theta$

4. William put the tip of his pencil on the outer edge of a graph of the unit circle at the point (0, -1). He moved this pencil tip through an angle of $\frac{4\pi}{3}$ radians in the counterclockwise direction along the edge of the circle. At what angle of the unit circle did William's pencil tip stop?

A. $\frac{\pi}{3}$ B. $\frac{5\pi}{6}$ C. $\frac{7\pi}{6}$ D. $\frac{5\pi}{3}$

$\frac{4\pi}{3}$ radians $= 240^0$
So if we start at (0,-1) and rotate 240^0 counterclockwise we land on $\frac{5\pi}{6}$

5. Which of the following functions is graphed below?

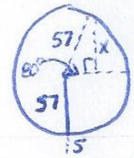


amplitude = 3

A. $y = 3\cos\theta$ B. $y = 3\sin\theta$ C. $y = \cos 3\theta$ D. $y = \sin 3\theta$

6. A Ferris wheel has a diameter of 114 feet and is 5 feet off the ground. After a person gets on the bottom car, the Ferris wheel rotates 170^0 counterclockwise before stopping. How high above the ground is the car when it has stopped?

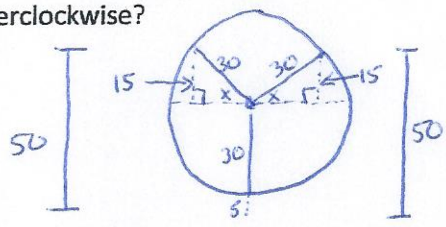
A. 56 feet B. 62 feet C. 80 feet D. 118 feet



$\sin(80) = \frac{x}{57}$
$x = 56.13$

$57 + 56.13 + 5 = 118.13$

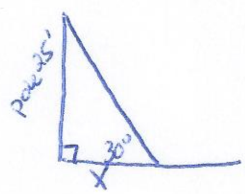
7. A Ferris wheel has a radius of 30 meters and is 5 meters off the ground. If a person on the Ferris wheel is 50 meters above the ground, at what degree(s) had the Ferris wheel rotated counterclockwise?



$\sin(x) = \frac{15}{30}$
$\sin^{-1}(\frac{15}{30}) = 30$
So either 120° or 240^0

8. A rope is attached to the top of a 25-foot pole. The pole is perpendicular to the ground. Approximately how far from the base of the pole must the rope be attached to make a 30^0 angle with the ground?

A. 12.5 feet B. 14.4 feet C. 43.3 feet D. 50.0 feet

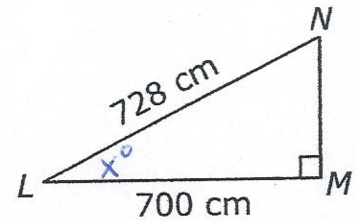


$\tan(30) = \frac{25}{x}$

$x \cdot \tan(30) = 25$

$x = \frac{25}{\tan(30)} = 43.3$

9. In right triangle LMN, $LN = 728$ cm and $LM = 700$ cm.



What is the ***approximate*** measure of $\angle NLM$?

A 15.9°

B 16.6°

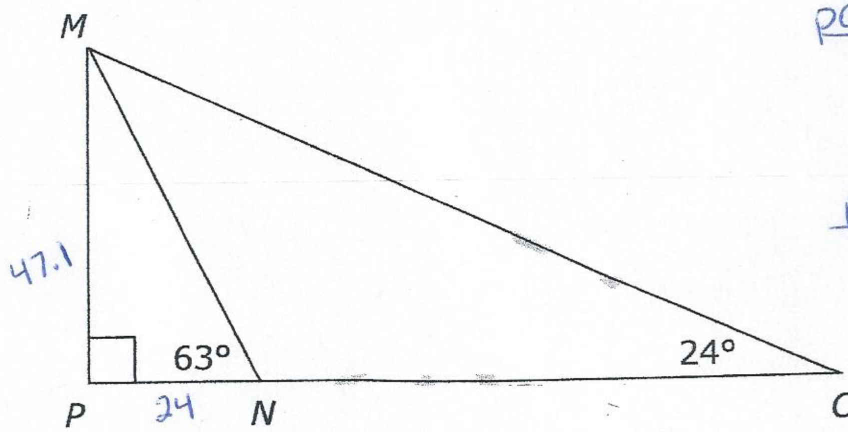
C 73.4°

D 74.1°

$\cos(x) = \frac{700}{728}$

$\cos^{-1}\left(\frac{700}{728}\right) = 15.9$

10. In the diagram below, Triangle MPO is a right triangle and $\overline{PN} = 24$ ft.



- What is the length of $\overline{MP}$?
- How much longer is $\overline{MO}$ than $\overline{NM}$?
- How far is point O from point N?

part 1 $\tan(63) = \frac{x}{24}$

$x = 24 \cdot \tan(63)$

$x = 47.1$ $\overline{MP} = 47.1$

part 2

$\overline{MN}$ $\cos(63) = \frac{24}{x}$

$x \cdot \cos(63) = 24$

$x = \frac{24}{\cos(63)} = 52.86$

$\overline{MN} = 52.86$

$\overline{MO}$ $\sin(24) = \frac{47.1}{x}$

$x \cdot \sin(24) = 47.1$

$x = 115.8$

$\overline{MO} = 115.8$

$115.8 - 52.86 = 62.94$

part 3

$\tan(24) = \frac{47.1}{x}$

$x \cdot \tan(24) = 47.1$

$x = \frac{47.1}{\tan(24)} = 105.79$

$105.79 - 24 = 81.79$

11. Which expression is equivalent to $\frac{sin^4(\theta)-cos^4(\theta)}{sin^2(\theta)-sin^2(\theta)}$, where $sin^2(\theta) \neq cos^2(\theta)$?

A. $sin^2(\theta) - cos^2(\theta)$

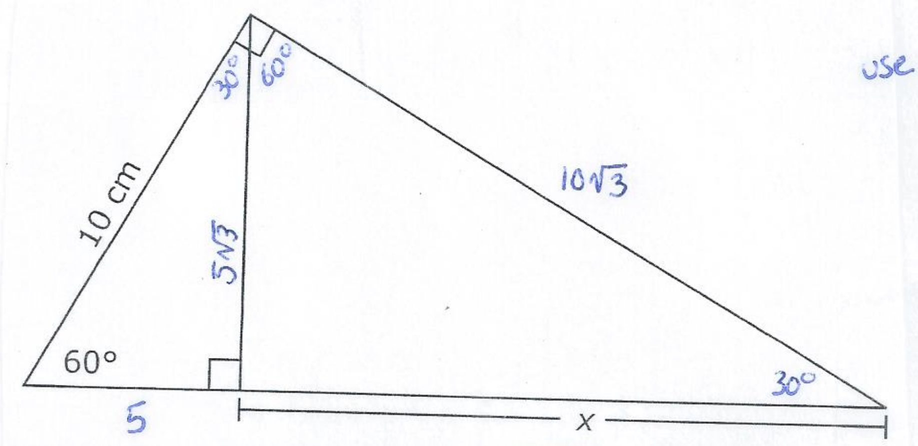
B. $cos^2(\theta) - sin^2(\theta)$

C. 2

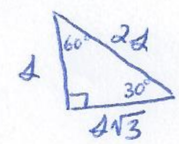
D. 1

$$\frac{(\sin^2\theta + \cos^2\theta)(\cancel{\sin^2\theta - \cos^2\theta})}{\cancel{\sin^2\theta - \sin^2\theta}} = \frac{\sin^2\theta + \cos^2\theta}{1} = \frac{1}{1} = 1$$

12. What is the value of x in the triangle below?



use 30-60-90 rule



$5\sqrt{3}\sqrt{3}$

$5\sqrt{9}$

$5(3) = 15$

A. $\frac{5\sqrt{3}}{2}$ cm B. $5\sqrt{3}$ cm C. 10 cm D. 15 cm

13. Which angle, in standard position, is NOT coterminal with the others?

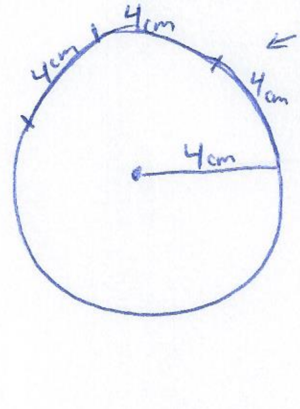
A. -190^0 B. -170^0 C. 190^0 D. 550^0

$r=4$

14. The diameter of a circle is 8 centimeters. A central angle of the circle intercepts an arc of 12 centimeters. What is the radian measure of the angle?

A. $\frac{3}{2}$ B. 3 C. 4 D. 8π

← takes 3 radians to get to 12cm



120°

15. In a circle, an arc of length 8π cm is intercepted by a central angle of $\frac{2\pi}{3}$ radians. What is the radius of the circle?

$\frac{120}{360} \cdot 2\pi r = 8\pi$

$\left(\frac{3}{2}\right)\frac{2}{3}\pi r = 8\pi\left(\frac{3}{2}\right)$

$\pi r = 12\pi$

$r = 12$

A. $\frac{3\pi}{16}$ cm B. $\frac{16\pi}{3}$ cm C. $\frac{16\pi^2}{3}$ cm D. 12 cm

16. What is the amplitude of $y = 3\sin 4\theta$?

A. $\frac{4}{3}$ B. 3 C. 4 D. 2π

17. Which answer choice describes $y = -\sin 2\theta$?

A. amplitude -1, period 4π B. amplitude 1, period π

C. amplitude 2, period $-\pi$ D. amplitude 2π, period 1

18. Which function has a period of 4π and an amplitude of 8?

A. $y = -8\sin 8\theta$ B. $y = -8\sin\frac{1}{2}\theta$ C. $y = 8\sin 2\theta$ D. $y = 4\sin 8\theta$

19. Which function is a phase shift of $y = \sin\theta$ by 5 units to the left?

A. $y = 5\sin\theta$ B. $y = \sin\theta + 5$ C. $y = \sin(\theta + 5)$ D. $y = \sin 5\theta$