Honors Math 3

Name: Key

Polynomial Practice Problems

Write each polynomial function in standard form. Then determine the end behavior of each.

1. $n = 4m^2 - m + 7m^4$

$n = 7m^4 + 4m^2 - m$

As $m \to -\infty$, $n \to \infty$

As $m \to \infty$, $n \to \infty$

2. $f(t) = 4t + 3t^3 + 2t - 7 = 3t^3 + 6t - 7$

As $t \to -\infty$, $f(t) \to -\infty$

As $t \to \infty$, $f(t) \to \infty$

3. $f(r) = 5r + 7 + 2r^2 = 2r^2 + 5r + 7$

As $r \to -\infty$, $f(r) \to \infty$

As $r \to \infty$, $f(r) \to \infty$

Divide using long division for #s 4 and 5:

4. $\frac{x^4 - 23x^3 + 49x + 4}{x^2 + x - 2}$

$x^2 - 24x + 26$

$x^2 + x - 2 \overline{)\, x^4 - 23x^3 + 0x^2 + 49x + 4}$

$-(x^4 + x^3 - 2x^2)$

$-24x^3 + 2x^2 + 49x + 4$

$-(-24x^3 - 24x^2 + 48x)$

$26x^2 + x + 4$

$-(26x^2 + 26x - 52)$

$-25x + 56$

$= x^2 - 24x + 26 + \frac{-25x + 56}{x^2 + x - 2}$

5. $(2x^3 - 6x^2 + 4x + 1) \div (x^2 + 3)$

$2x - 6$

$x^2 + 0x + 3 \overline{)\, 2x^3 - 6x^2 + 4x + 1}$

$-(2x^3 + 0x^2 + 6x)$

$-6x^2 - 2x + 1$

$-(-6x^2 + 0x - 18)$

$-2x + 19$

$= 2x - 6 + \frac{-2x + 19}{x^2 + 3}$

Divide using synthetic division for #s 6 and 7:

6. $(2x^3 - 3x^2 - 18x - 8) \div (x - 4)$

4	2	-3	-18	-8
	↓	8	20	8
	2	5	2	0

$2x^2 + 5x + 2$

7. $(6x^3 - x^2 + 8) \div (x + 2)$

-2	6	-1	0	8
	↓	-12	26	-52
	6	-13	26	-44

$6x^2 - 13x + 26 + \frac{-44}{x + 2}$

Find all solutions of each equation.

8. $x^4 + 14x^2 - 32 = 0$

$-32 \mid 14$

$16 \mid -2$

$x^4 + 16x^2 - 2x^2 - 32 = 0$

$x^2(x^2 + 16) - 2(x^2 + 16) = 0$

$(x^2 - 2)(x^2 + 16) = 0$

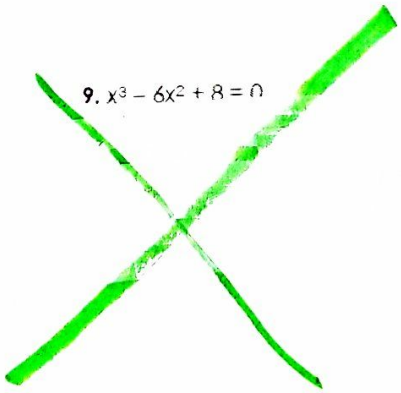
$x^2 - 2 = 0$ $\quad x^2 + 16 = 0$

$x^2 = 2$ $\quad x^2 = -16$

$x = \pm\sqrt{2}$ $\quad x = \pm 4i$

$x = \{\pm\sqrt{2}, \pm 4i\}$

9. $x^3 - 6x^2 + 8 = 0$



10. $6x^3 - 2x^2 + 4x = 0$

x-int @ $x = 0$

0	6	-2	4	0
	↓	0	0	0
	6	-2	4	0

$6x^2 - 2x + 4 = 0$

$x = \frac{2 \pm \sqrt{4 - 4(6)(4)}}{2(6)} = \frac{2 \pm \sqrt{-92}}{12}$

$x = \frac{2 \pm 2i\sqrt{23}}{12} = \frac{1 \pm i\sqrt{23}}{6}$

$x = \left\{0, \frac{1 \pm i\sqrt{23}}{6}\right\}$

11. $x^4 - 5x^3 - 8x = -40$

$x^4 - 5x^3 - 8x + 40 = 0$

x-int @ $x = 2, 5$

2	1	-5	0	-8	40
	↓	2	-6	-12	-40
5	1	-3	-6	-20	0
	↓	5	10	20	
	1	2	4	0	

$x^2 + 2x + 4 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4(1)(4)}}{2(1)} = \frac{-2 \pm \sqrt{-12}}{2}$

$x = \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$

$x = \{2, 5, -1 \pm i\sqrt{3}\}$

12. $x^3 + 125 = 0$

$(x + 5)(x^2 - 5x + 25) = 0$

$x + 5 = 0$ $\quad x^2 - 5x + 25 = 0$

$x = -5$ $\quad x = \frac{5 \pm \sqrt{25 - 4(1)(25)}}{2(1)}$

$x = \frac{5 \pm \sqrt{-75}}{2} = \frac{5 \pm 5i\sqrt{3}}{2}$

$x = \left\{-5, \frac{5 \pm 5i\sqrt{3}}{2}\right\}$

13. $x^3 - 3x^2 + 4x - 12 = 0$

$x^2(x - 3) + 4(x - 3) = 0$

$(x^2 + 4)(x - 3) = 0$

$x^2 + 4 = 0$ $\quad x - 3 = 0$

$x^2 = -4$ $\quad x = 3$

$x = \pm 2i$

$x = \{3, \pm 2i\}$

14. $3x^4 + 11x^3 + 14x^2 + 7x + 1 = 0$

-1	3	11	14	7	1
	↓	-3	-8	-6	-1
-1	3	8	6	1	0
	↓	-3	-5	-1	
	3	5	1	0	

$3x^2 + 5x + 1$

$x = \frac{-5 \pm \sqrt{25 - 4(3)(1)}}{2(3)} = \frac{-5 \pm \sqrt{13}}{6}$

$x = \left\{-1 \text{ mult } 2, \frac{-5 \pm \sqrt{13}}{6}\right\}$

15. $3x^4 - x^3 - 22x^2 + 24x = 0$

-3	3	-1	-22	24	0
	↓	-9	30	-24	0
0	3	-10	8	0	0
	↓	0	0	0	
2	3	-10	8	0	
	↓	6	-8		
	3	-4	0		

$3x - 4 = 0 \to 3x = 4 \to x = 4/3$

$x = \{-3, 0, 2, 4/3\}$

16. $45x^3 + 93x^2 - 12 = 0$

$x = \{-2, 1/3, -2/5\}$

-2	45	93	0	-12
	↓	-90	-6	12
	45	3	-6	0

$45x^2 + 3x - 6 = 0$ $\quad \div 3$

$15x^2 + x - 2 = 0$

$15x^2 + 6x - 5x - 2 = 0$

$3x(5x + 2) - 1(5x + 2) = 0$

$(3x - 1)(5x + 2) = 0$

$-30 \mid 1 \mid -5$

17. What is $P(-4)$ given that $P(x) = 2x^4 - 3x^3 + 5x^2 - 1$? Show using **Synthetic Division.**

$$\begin{array}{c|ccccc} -4 & 2 & -3 & 5 & 0 & -1 \\ & \downarrow & -8 & 44 & -196 & 784 \\ \hline & 2 & -11 & 49 & -196 & 783 \end{array}$$

$P(-4) = 783$

18. Write the equation of a polynomial function that has zeros at:

- -3 and $2-i$.

$2+i$

$(x+3)(x-2-i)(x-2+i) = (x+3)(x^2-4x+5)$

	x^2	$-2x$	ix
-2	$-2x$	4	$-2i$
$-i$	$-ix$	$2i$	1

$y = (x+3)(x^2-4x+5)$

- $3-i$ and 2

$3+i$

$(x-2)(x-3-i)(x-3+i) = (x-2)(x^2-6x+10)$

	x^2	$-3x$	ix
-3	$-3x$	9	$-3i$
$-i$	$-ix$	$3i$	1

$y = (x-2)(x^2-6x+10)$

- $2i$ and 3 and a double root at 4

$-2i$

$(x-4)^2(x-3)(x-2i)(x+2i) = (x-4)^2(x-3)(x^2+4)$

	x^2	$2ix$
$-2i$	$-2ix$	4

$y = (x-4)^2(x-3)(x^2+4)$

19. The remainder when dividing $x^3 + 2x^2 + 3x + k$ by $(x+1)$ is 2. What is the remainder when dividing the same polynomial by $(x-2)$?

$$\begin{array}{c|cccc} -1 & 1 & 2 & 3 & K \\ & \downarrow & -1 & -1 & -2 \\ \hline & 1 & 1 & 2 & 2 \end{array}$$

$K-2=2 \rightarrow K=4$

$$\begin{array}{c|cccc} 2 & 1 & 2 & 3 & 4 \\ & \downarrow & 2 & 8 & 22 \\ \hline & 1 & 4 & 11 & 26 \end{array}$$

Remainder is 26.

20. The volume of a box is $x^3 + 4x^2 + 4x$. What are the dimensions of the box?

$x(x^2+4x+4)$

$x(x+2)^2$

The length, width, and height are x, $x+2$, and $x+2$.

Use synthetic division and the given factor to completely factor the binomial function.

21. $f(x) = x^3 + 2x^2 - 5x - 6$; $(x+1)(x+3)(x-2)$

$$\begin{array}{c|cccc} -1 & 1 & 2 & -5 & -6 \\ & \downarrow & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x^2+x-6 \rightarrow (x+3)(x-2)$

22. $f(x) = x^3 + 9x^2 + 23x + 15$; $(x+5)(x+3)(x+1)$

$$\begin{array}{c|cccc} -5 & 1 & 9 & 23 & 15 \\ & \downarrow & -5 & -20 & -15 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$x^2+4x+3 \rightarrow (x+3)(x+1)$

23. Find the polynomial given the following

- Find a polynomial function whose graph passes through the points: (-3, -50), (-2, -4), (-1, 10), (0, 7), and (2, -23)

Cubic: $y = 1.617924528x^3 + -5.091644205x^2 + -11.04380054x + 6.555256065$

r^2 0.9994067684

Quartic: $y = -0.25x^4 + 0.85x^3 - 4.025x^2 - 8.15x + 7$

Best Fit

r^2 1

- Find a cubic and a quartic model for each set of values. Explain why one models the data better.

x	-2	-1	0	1	2
y	-65	-14	-4	2	90

Cubic: $y = 10.25x^3 + 5x^2 - 2.25x - 8.2$

r^2 0.997261433 3

Quartic: $y = 2.041\overline{6}x^4 + 10.25x^3 - 4.041\overline{6}x^2 - 2.25x - 4$

Best fit

r^2 1

24. Factor the following

$s^3 - 64$

$(s-4)(s^2+4s+16)$

$m^3 + 216$

$(m+6)(m^2-6m+36)$

$y^3 + 125$

$(y+5)(y^2-5y+25)$

$27x^3 - y^3$

$(3x-y)(9x^2+3xy+y^2)$

$125x^3 + 8a^3$

$(5x+2a)(25x^2-10ax+4a^2)$

$1000 + 27a^3$

$(10+3a)(100-30a+9a^2)$

$3a^3 - 81x^3 = 3(a^3-27x^3)$

$3(a-3x)(a^2+3ax+9x^2)$

$r^3 + 8b^3$

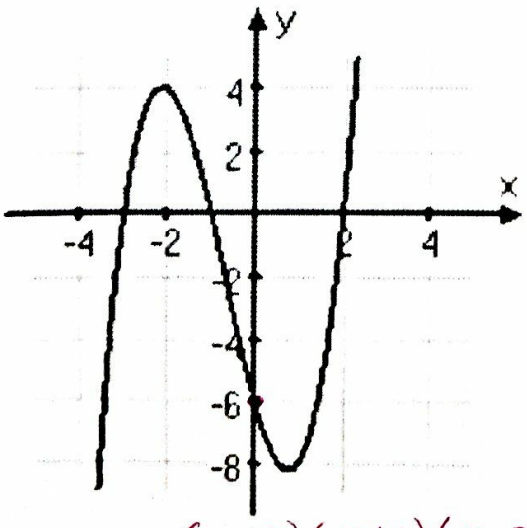
$(r+2b)(r^2-2br+4b^2)$

$(2x-3)^3 - y^3$

$((2x+3)-y)((4x^2+12x+9)+2xy+3y+y^2)$

$(2x-y+3)(4x^2+y^2+2xy+12x+3y+9)$

25. Write the equation given the following graphs:



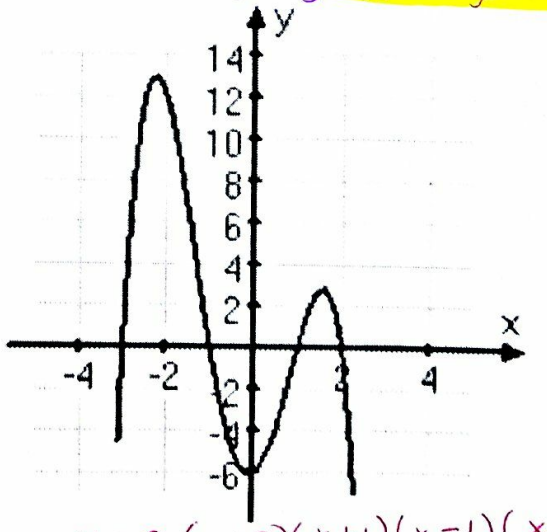
$y = a(x+3)(x+1)(x-2)$

$-6 = a(0+3)(0+1)(0-2)$

$-6 = -6a$

$1 = a$

$y = (x+3)(x+1)(x-2)$



$y = a(x+3)(x+1)(x-1)(x-2)$

$-6 = a(0+3)(0+1)(0-1)(0-2)$

$-6 = (3)(-1)(-2)a$

$-6 = 6a$

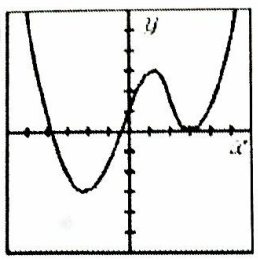
$-1 = a$

$y = -(x+3)(x+1)(x-1)(x-2)$

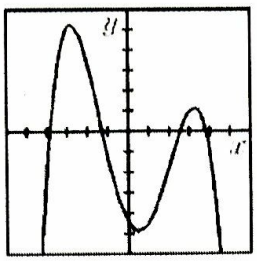
26.

Which of the following graphs best illustrates the graph of $y = a(x-b)(x-c)(x-d)^2$ where $a > 0$ and $b \neq c \neq d$?

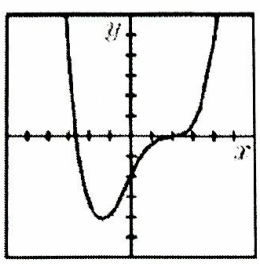
A.



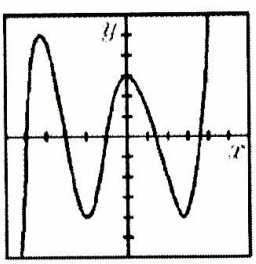
B.



C.

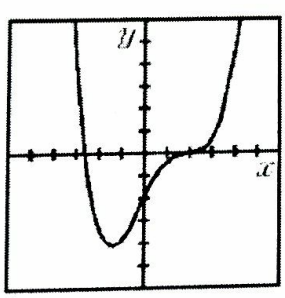


D.

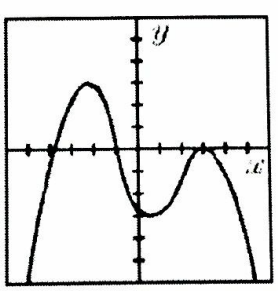


Which of the following graphs best illustrates the graph of $y = a(x-b)(x-c)(x-d)^2$ where $a < 0$ and $b \neq c \neq d$?

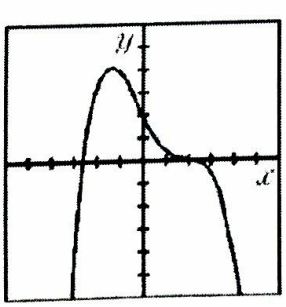
A.



B.



C.



D.