Name: Key

Algebra II, Period ___

* Skip: Power, roots, & radicals Function Operations

Date: _____

Math Department

* add: synthetic | long & one-to-one

Final Exam Review Packet - Algebra II

- This review packet contains questions that are similar to the type of problems that you will encounter on the exam.
- The in-class review is not meant to re-teach you everything from the second semester. It will be a quick, but thorough overview of the material.
- It is recommended that you work on this review packet leading up to your exam day so you have questions ready. Don't wait till the last minute.
- Remember that the exam counts for 20% or your course grade.
- Reviewing for the exam is **YOUR** responsibility.
- If you have questions as you prepare, make arrangements to see your teacher.

I. Quadratics Equations

Solve each of the following equations using factoring.

a. $x^2-36=0$

$(x+6)(x-6)=0$

$x=-6,6$

b. $7x^2-14x=0$

$7x(x-2)=0$

$7x=0 \quad x-2=0$

$x=0,2$

c. $x^3-6x^2-7x=0$

$x(x^2-6x-7)=0$

$x(x-6)(x+1)=0$

$x=0,6,-1$

d. $6x^2+7x-3=0$

-18, $9 \cdot -2$, -1

$(6x^2+9x)(-2x-3)=0$

$3x(2x+3)-1(2x+3)=0$

$(3x-1)(2x+3)=0$

$x=1/3, -3/2$

e. $3x^2+3x-36=0$

$3(x^2+x-12)=0$

$3(x+4)(x-3)=0$

$x=-4,3$

f. $32x^2-2=0$

$2(16x^2-1)=0$

$2(4x+1)(4x-1)$

$x=\pm\frac{1}{4}$

g. $(x^3-2x^2)+(-9x+18)=0$

$x^2(x-2)-9(x-2)=0$

$(x^2-9)(x-2)=0$

$(x+3)(x-3)(x-2)=0$

$x=2,\pm 3$

h. $(x^3-3x^2)+(6x-18)=0$

$x^2(x-3)+6(x-3)=0$

$(x^2+6)(x-3)=0$

$x=3,\pm i\sqrt{6}$

Quadratics Equations (continued)

Factor each polynomial COMPLETELY.

Sum of Two Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of Two Cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

a. x^3+27 SOAP

$(x+3)(x^2+3x+9)$

b. $8x^3-125$

$(2x-5)(4x^2+10x+25)$

c. x^4+5x^2-14

$(x^2+7)(x^2-2)$

-14: 7, -2; 5

d. $2x^5-18x^3+40x$

$2x(x^5-9x^2+20)$

$2x(x^2-5)(x^2-4)$

$2x(x^2-5)(x+2)(x-2)$

20: -5, -4; -9

Solve each of the following equations using the Quadratic Formula.

Quadratic Formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

a. $4x^2+6x+1=0$

b. $x^2+2x+2=0$

c. $2x^2+3x-5=0$

d. $3x^2-2x-7=0$

III. Simplifying Rational Expressions

Simplify the Rational Expression using Multiplication or Division.

a. $\frac{x^2+4x-12}{x^2(x^2+9x+18)}\cdot 6x^2$

$\frac{(x+6)(x-2)}{x^2(x+6)(x+3)}\cdot 6x^2 = \frac{6(x-2)}{(x+3)}$

$x \neq -6, -3, 0$

f. $\frac{12x^2y^3z}{6x^3y^2z^2}$

$= \frac{2y}{xz}$

b. $\frac{3x^2-12}{5x-10}\cdot\frac{1}{2x+4}$

$\frac{3(x^2-4)}{5(x-2)}$

$\frac{3(x+2)(x-2)}{5(x-2)}\cdot\frac{1}{2(x+2)} = \frac{3}{10}$

$x \neq 2, -2$

g. $\frac{x^3+3x^2}{2x}\div\frac{x^2+5x+6}{5x^3}$

$\frac{x^2(x+3)}{2x}\cdot\frac{5x^3}{(x+3)(x+2)}$

$\frac{5x^4}{2(x+2)}$ $x \neq -2, -3, 0$

c. $\frac{x^2-4}{x^2+4}\cdot\frac{x+2}{x-2}$

$\frac{(x+2)(x-2)}{x^2+4}\cdot\frac{x+2}{x-2} =$

$x \neq 2, \pm 2i$ $\frac{x^2+4x+4}{x^2+4}$

h. $\frac{x^2+x-20}{x+1}\div\frac{11x+55}{x+1}$

$\frac{(x+5)(x-4)}{(x+1)}\cdot\frac{(x+1)}{11(x+5)} = \frac{x-4}{11}$

$x \neq -5, -1$

d. $\frac{5x^2-20}{25x^2}\cdot\frac{x}{x-2}$

$5(x^2-4)$

$\frac{5(x+2)(x-2)}{25x^2}\cdot\frac{x}{x-2} = \frac{x+2}{5x}$

$5x$

$x \neq 0, 2$

i. $\frac{x^2+5x+6}{x+3}\div\frac{x^2-4}{x+1}$

$\frac{(x+3)(x+2)}{x+3}\cdot\frac{x+1}{(x+2)(x-2)} =$

$\frac{x+1}{x-2}$ $x \neq 2, -3, -2, -1$

e. $\frac{x^2+x-30}{1}\cdot\frac{x}{x^2+6x}$

$\frac{(x+6)(x-5)}{1}\cdot\frac{x}{x(x+6)}$

$x-5$

$x \neq 0, -6$

j. $\frac{x^2+6x-7}{3x^2}\div\frac{x+7}{6x}$

$\frac{(x+7)(x-1)}{3x^2}\cdot\frac{6x}{x+7} =$

$\frac{2(x-1)}{x}$

$x \neq 0, -1$

Simplifying Rational Expressions (continued)

Simplify the Rational Expression using Addition or Subtraction. (LCD = ?)

a. $\frac{5}{3x^2}+\frac{2}{5x}$ $\frac{4}{5}\frac{3x}{3x}$

$\frac{20}{15x^2}+\frac{6x}{15x^2}=\boxed{\frac{20+6x}{15x^2}}$

$x\neq 0$

---

b. $\frac{3}{2x-2}+\frac{x+1}{4}$

$2(x-1)$

$\frac{2}{2}\frac{3}{2(x-1)}+\frac{x+1}{4}\frac{(x-1)}{(x-1)}$ $\frac{6}{4(x-1)}+\frac{x^2-1}{4(x-1)}=\boxed{\frac{x^2-5}{4(x-1)}}$

$x\neq 1$

---

c. $\frac{4}{3x^3}+\frac{x}{6x^3+3x^2}$

$\left(\frac{2x+1}{2x+1}\right)\frac{4}{3x^3}+\frac{x}{3x^2(2x+1)}\left(\frac{x}{x}\right)=\frac{8x+4+x^2}{3x^3(2x+1)}=\boxed{\frac{x^2+8x+4}{3x^3(2x+1)}}$

$x\neq -\frac{1}{2}, 0$

---

d. $\frac{5x-1}{x^2+2x-8}-\frac{6}{x+4}$

FOIL

$\frac{5x-1}{(x+4)(x-2)}-\frac{6}{x+4}\left(\frac{x-2}{x-2}\right)=\frac{5x-1-6x+12}{(x+4)(x-2)}=\boxed{\frac{-x+11}{(x+4)(x-2)}}$

$x\neq -4, 2$

---

e. $\frac{x+1}{x^2+6x+9}-\frac{1}{x^2-9}$

$\frac{(x-3)(x+1)}{(x-3)(x+3)(x+3)}-\frac{1}{(x+3)(x-3)(x+3)}=\frac{x^2-2x-3-x-3}{(x+3)(x+3)(x-3)}$

$x\neq 3, -3$

$\boxed{\frac{x^2-3x-6}{(x+3)(x+3)(x-3)}}$

IV. Solving Rational Equations

Solve each rational equation.

a. $\frac{3}{x+4}=\frac{9}{x-2}$

$3(x-2)=9(x+4)$

$3x-6=9x+36$

$-6x=42$

$\boxed{x=-7}$

$x\neq -4, 2$

---

b. $\frac{4x}{x-1}=\frac{x}{x^2-1}$

$4x(x+1)=x$

$4x^2+4x=x$

$4x^2+3x=0$

$x(4x+3)=0$

$\boxed{x=0,\ -\frac{3}{4}}$

$x\neq 1, -1$

---

c. $\frac{3}{x^2-4}=\frac{2}{x+2}+\frac{x}{x-2}$

$3=2(x-2)+x(x+2)$

$2x-4+x^2+2x$

$3=x^2+4x-4$

$-3\quad -3$

$0=x^2+4x-7$

$x=\frac{-4\pm\sqrt{44}}{2}=\frac{-4\pm 2\sqrt{11}}{2}=$

$\boxed{-2\pm\sqrt{11}}$

$x\neq 2, -2$

---

d. $\frac{3x-2}{x-2}=\frac{6}{x^2-4}+1$

$\frac{3x-2(x+2)}{(x-2)(x+2)}=\frac{6}{(x+2)(x-2)}+\frac{1}{1}\frac{(x+2)(x-2)}{(x+2)(x-2)}$

$2x^2+4x-6=0$ Factor!

$(2x-2)(x+3)$

$\boxed{x=1, -3}$

$x\neq 2, -2$

---

e. $\frac{x}{x+2}=\frac{3x+1}{x-1}+\frac{4}{x^2+x-2}$

$x(x-1)=3x+1(x+2)+4$

$2x^2+8x+6=0$ Factor!

$(2x+2)(x+3)=0$

$\boxed{x=-1, -3}$

$x\neq -2, 1$

VI. Inverses

Find the inverse of each function.

a. $f(x)=2x+5$

$y=2x+5$

$x=2y+5$

$x-5=2y$

$\frac{x-5}{2}=\frac{2y}{2}$

$f^{-1}(x)=\frac{x-5}{2}$

b. $f(x)=\sqrt[3]{2x+4}$

$y=\sqrt[3]{2x+4}$

$(x)^3=\sqrt[3]{2y+4}^3$

$x^3=2y+4$

$\frac{x^3-4}{2}=\frac{2y}{2}$

$y=\frac{x^3-4}{2}$

c. $f(x)=5-\frac{5}{2}x$

$y=5-\frac{5}{2}x$

$x=5-\frac{5}{2}y$

$x-5=-\frac{5}{2}y$

$\frac{x-5}{-5}=\frac{-\frac{5}{2}y}{-\frac{5}{2}}$

$y=\frac{x-5}{1}\cdot\frac{2}{-5}$

d. $f(x)=\frac{x-2}{4}$

$y=\frac{x-2}{4}$

$4x=y-2$

$y=4x+2$

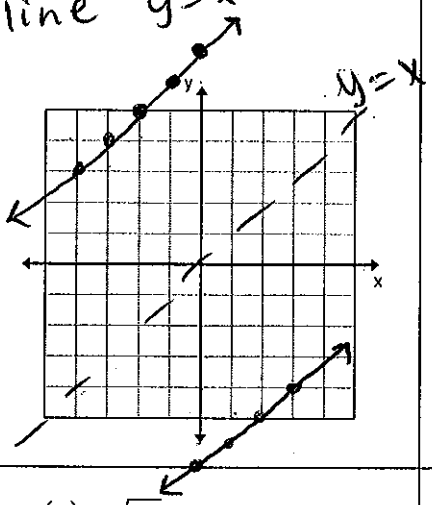
$f^{-1}(x)=4x+2$

Verify that the two functions are inverses of each other using composite functions. Then, verify (a) and (b) by graphing.

a. $f(x)=x+7,\ g(x)=x-7$

$f^{-1}(x)=\frac{2x-10}{-5}$

yes, reflected about the line $y=x$



b. $f(x)=\frac{1}{2}x+1,\ g(x)=2x-2$ ✓

$y=\frac{1}{2}x+1$

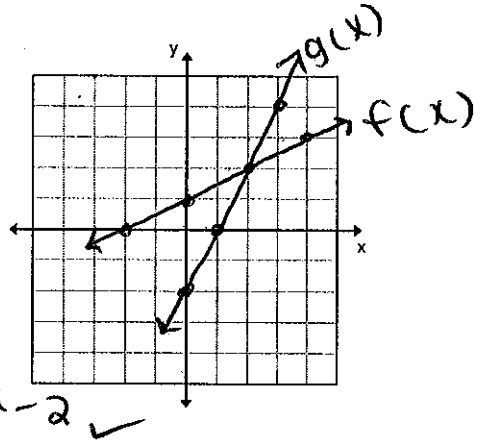
$x=\frac{1}{2}y+1$

$\frac{x-1}{\frac{1}{2}}=\frac{\frac{1}{2}y}{\frac{1}{2}}$

$2x-2=y$

$f^{-1}(x)=2x-2$ ✓

yes



c. $f(x)=\frac{1}{3}x^2,\ g(x)=\sqrt{3x}$

$y=\frac{1}{3}x^2$ ✓

$3\left(x=\frac{1}{3}y^2\right)$

$\sqrt{3x}=\sqrt{y^2}$

d. $f(x)=\frac{x^5+2}{7},\ g(x)=\sqrt[5]{7x-2}$

$y=\frac{x^5+2}{7}$

$x=\frac{y^5+2}{7}$

$7x=y^5+2$

$\sqrt[5]{7x-2}=\sqrt[5]{y^5}$

$y=\sqrt[5]{7x-2}$ ✓

The graph of the inverse function is the reflection of the original function over what line?

$y=x$

$y=\sqrt{3x}$

VII. Exponential & Logarithmic Functions

Solve each equation.

$y = \log_b x$ if and only if $x = b^y$.

Think of $y = \log_b x$ as the answer to: "To what power must b be raised to obtain x?"

a. $\left(\frac{1}{3}\right)^x = 27$

$x \log \frac{1}{3} = \log 27$

$x = \frac{\log 27}{\log \frac{1}{3}} = -3$

b. $5^{3x} = 25^{x-1}$

$5^{3x} = 5^{2(x-1)}$

$3x = 2x - 2$

$x = -2$

c. $4^x = 0.25$

$x \log 4 = \log .25$

$x = \frac{\log .25}{\log 4} = -1$

d. $10^x = 15$

$x \log 10 = \log 15$

$x = \frac{\log 15}{\log 10} = 1.2$

e. $e^{3x} = 24$

$\ln e^{3x} = \ln 24$

$3x = 3.18$

$x \approx 1.06$

f. $\ln 3x = -0.5003$

$e^{\ln 3x} = e^{-0.5003}$

$\frac{3x}{3} = \frac{.606}{3}$

$x \approx .202$

* g. $\log_x 64 = \frac{1}{2}$

$\frac{\log 64}{\log x} = .5$

$\frac{\log 64}{.5} = \log x$

$3.6 = \log x$

$\log_{10} x = 3.6$

$10^{3.6} = x$

$x = 4096$

h. $\log_3 x = 5$

$3^5 = x$

$243 = x$

i. $\log_4 256 = x$ → calculator!

$x = 4$

j. ~~$\log_7(2x+5) = \log_7(x-3)$~~

$2x + 5 = x - 3$

$-x \quad -5 \quad -x \quad -5$

$x = -8$

k. $\log_2(2x^2) = 5$

$2^5 = 2x^2 \quad x = \pm 4$

$32 = 2x^2$

$\sqrt{16} = \sqrt{x^2}$

l. $\log x = 2.096910013$

$10^{2.096910013} = x$

$125 = x$

m. $256e^{2x} = 1400$

$\ln e^{2x} = \ln 5.469$

$2x = 1.7$

$x \approx .85$

n. $75 = 21(1.05)^t$

$3.571428 = 1.05^t$

$\log 3.571428 = t \log 1.05$

$t = \frac{\log 3.571428}{\log 1.05}$

$t \approx 26.09$

o. $10^{x^2+3x-7} = 1{,}000$

$10^{x^2+3x-7} = 10^3$

$x^2 + 3x - 7 = 3$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5, 2$

Write the logs in condensed form.

a. $2\log x - x\log y$

$\log x^2 - \log y^x$

$\log \frac{x^2}{y^x}$

Write the logs in expanded form.

b. $\log x^2 y^3 z^4$

$2\log x + 3\log y + 4\log z$

c. $\log x + 2\log y$

$\log xy^2$

d. $\log(x^2+1)z$

$\log(x^2+1) + \log z$

e. $\log x + \frac{1}{2}\log y - 2\log z$

$\log x\sqrt{y}z^2$

f. $\log \frac{x^2}{z^6}$

$2\log x - 6\log z$

g. $\log x + \log y + \log z - 2\log w$

$\log \frac{xyz}{w^2}$

h. $\log x^2 y$

$2\log x + \log y$

* look over growth, decay, half-life too!

Exponential & Logarithmic Functions (continued)

Use the equation given and the properties of logs to solve the problems below:

(1) $A = P\left(1+\frac{r}{n}\right)^{nt}$

(2) $A = Pe^{rt}$

where:
- P = original amount deposited or the initial investment
- r = the interest rate expressed as a decimal (5% → 0.05)
- n = the number of times a year the interest is paid ("quarterly" → means n = 4)
- t = the number of years the investment spans

a. Find the value of a $1,000 investment at 6% interest after 10 years compounded:

(a) annually

$A = 1000\left(1+\frac{.06}{1}\right)^{10} \approx \$1,790.85$

(b) quarterly

$A = 1000\left(1+\frac{.06}{4}\right)^{(4)(10)} \approx \$4814.02$

(c) monthly

$A = 1000\left(1+\frac{.06}{12}\right)^{(12)(10)} \approx \$1,819.397$

(d) continuously

$A = Pe^{rt}$

$1000e^{.06(10)} \approx 1,822.12$

b. If you invest $30,000 at 4.76% interest paid quarterly, how long would it take you to double your money? Round your answer to the nearest hundredth.

$60,000 = 30,000\left(1+\frac{.0476}{4}\right)^{4t}$

$2 = 1.0119^{4t}$

$\log 2 = \log 1.0119^{4t}$

$\log 2 = 4t \cdot \log 1.0119$

$\frac{\log 2}{\log 1.0119} = \frac{4t \cdot \log 1.0119}{\log 1.0119}$

$58.59 = 4t \rightarrow t = 14.65$ years

c. Suppose $2,000 is invested in a 3-year certificate of deposit (CD) that earns 6% interest, compounded continuously. How much will the investment be worth after 3 years?

$A = Pe^{rt}$

$A = 2000e^{(.06)(3)}$

$A = \$2,394.43$

d. You invest $200 at 12.25% earning continuous interest. How many years does it take for your money to increase 5 times its original value? Round your answer to the nearest tenth.

$1000 = 200e^{(.1225)t}$

$5 = e^{.1225t}$

$\ln 5 = \ln e^{.1225t}$

$\frac{1.609}{.1225} = \frac{.1225t}{.1225}$

$t \approx 13.14$ years