15. Given $s = 44.72$ feet and $r = 12.83$ feet, find θ. Round to the nearest hundredth.

A 573.76 radians

B 3.49 radians

C 0.29 radians

D 570.27 radians

(DOK 1)

16. Given $\theta = 4.64$ radians and $r = 8.39$ meters, find s. Round to the nearest hundredth.

A 0.55 meters

B 1.81 meters

C 37.12 meters

D 38.93 meters

(DOK 1)

17. Find sin A if angle A lies in quadrant IV and $\cos A = \frac{1}{4}$.

A $\sin A = \frac{\sqrt{15}}{4}$

B $\sin A = -\frac{\sqrt{15}}{4}$

C $\sin A = 4$

D $\sin A = -4$

(DOK 1)

18. Find cos B if angle B lies in quadrant III and $\tan B = 2$.

A $\cos B = -1$

B $\cos B = 1$

C $\cos B = -\frac{\sqrt{5}}{5}$

D $\cos B = \frac{\sqrt{5}}{5}$

(DOK 1)

19. Find tan C if angle C is acute and $\sec C = 2$.

A $\tan C = \frac{\sqrt{1}}{2}$

B $\tan C = -\frac{\sqrt{1}}{2}$

C $\tan C = -\sqrt{3}$

D $\tan C = \sqrt{3}$

(DOK 1)

20. Find csc D if angle D is acute and $\tan D = 1$.

A $\csc D = \sqrt{2}$

B $\csc D = -\sqrt{2}$

C $\csc D = \frac{\sqrt{2}}{2}$

D $\csc D = -\frac{\sqrt{2}}{2}$

(DOK 1)

Chapter 16
Combining and Comparing Functions

This chapter covers the following Common Core Standard(s):

	Content Standards:
Reasoning with Equaitons and Inequalities	HSA-REI.11
Interpreting Functions	HSF-IF.6, HSF-IF.9
Building Functions	HSF-BF.1b, HSF-BF.3

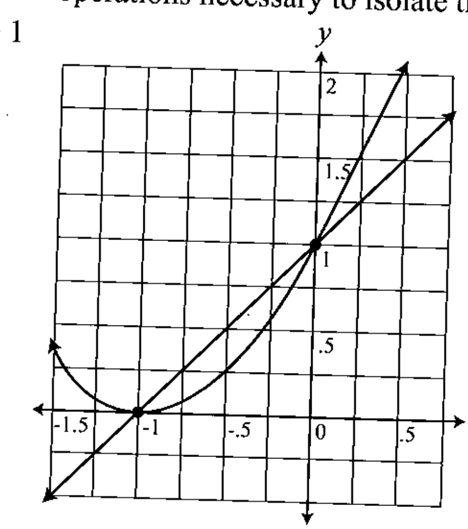
16.1 Solutions of Equations (DOK 2)

We have already discussed in a previous section what order of operations are necessary to isolating a variable when solving simple equations. We have not, however, practiced on equations that involve two functions set equal to each other. The idea behind solving these more complex equations is to find for what values they are equal. In other words, to solve for the values at which their graphs intersect.

Example 1: Determine at which points $f(x) = x^2 + 2x + 1$ and $g(x) = x + 1$ intersect. There are multiple ways to solve this: graphically, algebraically, or by making a table of values.

Step 1: Set the functions equal to each other.
$x^2 + 2x + 1 = x + 1$

Step 2: Perform the order of operations necessary to isolate the variable.
$x^2 + 2x + 1 = x + 1$
$x^2 + x = 0$
$x(x + 1) = 0$



Step 3: Recognize that this statement is true for both values $x = 0$ and $x = -1$. Therefore, 0 and -1 are the solutions to the equation.
Making a table:

$f(x) = x^2 + 2x + 1$

x	$f(x)$
-1	0
0	1
1	4
2	9
3	16

$g(x) = x + 1$

x	$g(x)$
-1	0
0	1
1	2
2	3
3	4

We can also see here that the values -1 and 0 are equal for both functions.

Answer: $x = -1, x = 0$

Determine where the graphs of the following functions intersect.

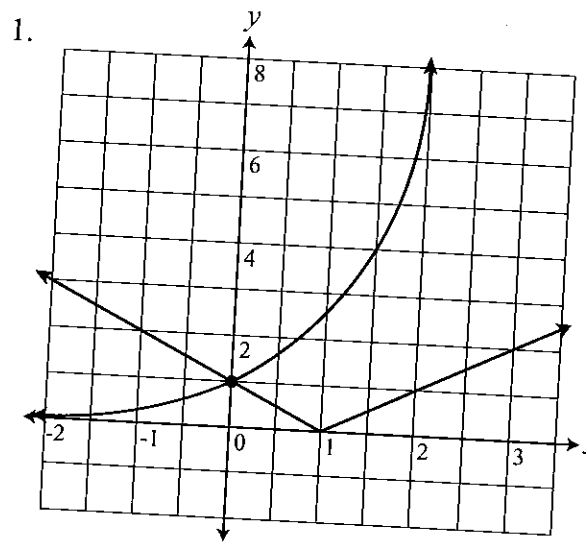
1. $f(x) = x^2 + 7x + 10,\ g(x) = 2x + 6$
2. $f(x) = 2^x,\ g(x) = e^{-x}$
3. $f(x) = |x|,\ g(x) = x^2 + 5x + 6$
4. $f(x) = \frac{1}{3}x + 2,\ g(x) = -x^2 + 2$
5. $f(x) = 3x + 4,\ g(x) = x^2 - 14$
6. $f(x) = 10^x - 99,\ g(x) = -x^2 + 5$
7. $f(x) = |x| - 10,\ g(x) = \frac{5}{4}x - 2$
8. $f(x) = 2^x,\ g(x) = \left(\frac{1}{2}\right)^x$
9. $f(x) = x^2 + 8x + 15,\ g(x) = -x^2 - x + 8$
10. $f(x) = x^2 + 2x - 35,\ g(x) = \left|\frac{80 - x}{2}\right|$
11. $f(x) = \log(x) + 5,\ g(x) = 5^x$
12. $f(x) = 5x^2 - 44x + 120,\ g(x) = 11x - 30$
13. $f(x) = 3(x - 2)^2,\ g(x) = \log(x) + 7$
14. $f(x) = x + 6,\ g(x) = 5x^2$
15. $f(x) = (x + 4)^2,\ g(x) = \log(x^2) + 10$

16.2 Finding Common Solutions of Functions (DOK 2)

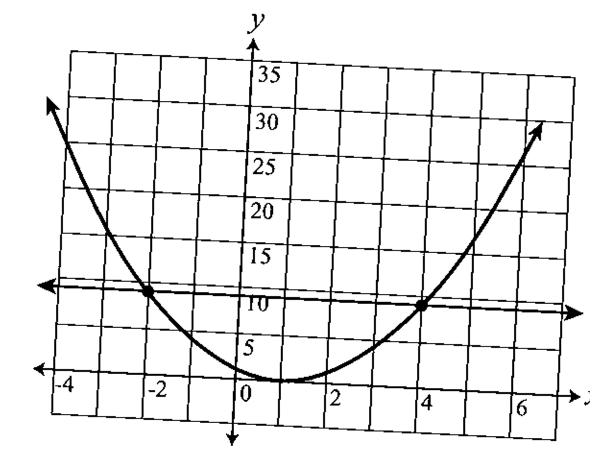
Now that we know how to solve for the solutions of equations algebraically, we will now solve for the solutions graphically, or using tables. Keep in mind that the solution to an equation is where the two graphs intersect.

For what value(s) of x are the following functions equivalent?

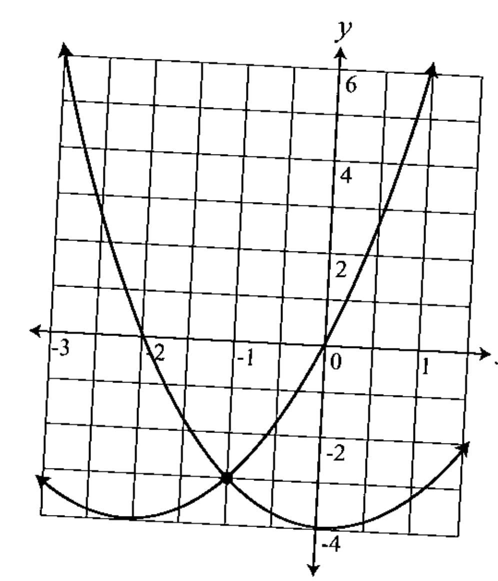
1.



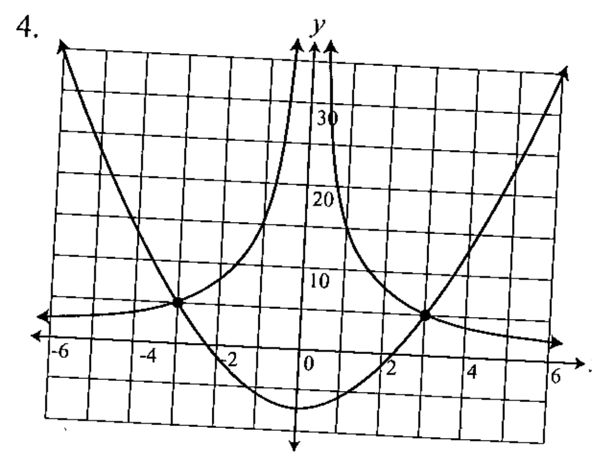
3.



2.



4.



5.

x	$f(x)$	$g(x)$
–2	4	2
–1	1	undefined
0	0	0
–1	1	$\frac{1}{2}$
2	4	$\frac{2}{3}$

7.

x	$f(x)$	$g(x)$
–4	7	7
–3	0	undefined
–2	–5	–5
–1	–8	–2
0	–9	–1
1	–8	$-\frac{1}{2}$
2	–5	$-\frac{1}{3}$
3	0	0

6.

x	$f(x)$	$g(x)$
–2	–18	10
–1	–13	8
0	–8	6
1	–3	4
2	2	2

8.

x	$f(x)$	$g(x)$
–4	–2	–2
–3	–2	0
–2	0	2
–1	4	4
0	10	6

16.3 Finding Rate of Change of a Function (DOK 2, 3)

To find the average rate of change of a function, $f(x)$ over a given interval $[a,b]$, we use the formula : $\frac{f(b)-f(a)}{b-a}$. We see that there is a similarity of this formula to the one we use to find the slope of a line: $\frac{y_2 - y_1}{x_2 - x_1}$. The difference is that a line has a constant rate of change, while other functions tend to increase or decrease over a certain interval. This is why the average rate of change may be different over two separate intervals of the same function.

Example 2: Find the average rate of change of the function $f(x) = x^2 + 3x + 2$ over the interval [0, 5].

Step 1: Determine the value of $f(0)$ and $f(5)$.

$f(0) = 0^2 + 3(0) + 2 = 0 + 0 + 2 = 2$

$f(5) = 5^2 + 3(5) + 2 = 25 + 15 + 2 = 42$

Step 2: Plug the values into the formula and simplify:

$$\frac{f(b) - f(a)}{b - a} = \frac{f(5) - f(0)}{5 - 0} = \frac{42 - 2}{5 - 0} = \frac{40}{5} = 8$$

Step 3: The average rate of change of the function $f(x)$ is 8.

Determine the average rate of change of the following functions over the given interval.

1. $f(x) = 2x^3 - 5x^2 + x - 7$, [3, 8]
2. $f(x) = 3x^3 - 6x^2 - 45x$, [7, 12]
3. $f(x) = 7 - 3x^2$, [2, 5]
4. $f(x) = 1.5^x$, [–1, 0]
5. $f(x) = 10x^2$, [6, 10]
6. $f(x) = 3x^2 - 2x + 1$, [0, 8]
7. $f(x) = x^3 - 3x + 4$, [1, 7]
8. $f(x) = 3x^3 - 5x^2 + 1$, [–1, 4]
9. $f(x) = x^4 - 5x$, [–2, 3]
10. $f(x) = x^2 + x - 2$, [2, 9]
11. On Monday, the price of gas was $3.34 per gallon. On Saturday, the price had risen to $4.12 per gallon. What is the average rate of change of the price of a gallon of gas from Monday to Saturday?
12. In 2003, the population of a town was 36,571 and rose to 72,636 by 2008. What was the average rate of change of the population from 2003 to 2008?
13. Greg delivers 24 ft^2 of tile for $320 and 55 ft^2 of tile for $630. What is the average rate of change of the cost as the number of square feet increases from 24 to 55?
14. An average 4-door sedan gets a fuel efficiency of 31 miles per gallon when driving at a speed of 60 mph. If the driver slows to a speed of 48 mph, the fuel efficiency increases to 37 miles per gallon. What is the average rate of change of the fuel efficiency as the speed drops from 60 mph to 48 mph?
15. Bill leaves on a road trip Sunday morning at 7:00 am and arrives at his destination at 2:00 pm. When he began his trip, the car's odometer read 19,772 miles and when he arrived it read 20,297 miles. What was his average speed for the trip?

16.4 Comparing Functions (DOK 3)

When two functions are each represented differently (algebraically, graphically, numerically in tables, or by verbal descriptions), it can be difficult to see their relationship with each other. This is why it is important to know how to identify functions and compare their properties in various ways.

Example 3: Which linear function below has a higher y-intercept?

$f(x) = -6x + 11$

x	$g(x)$
–2	–2
–1	3
0	8
1	13
2	18
3	23

Step 1: Identify the y-intercept. We know when looking at the algebraic representation of a linear function, that the value of the function when $x = 0$ is the y-intercept. Likewise when we look at a table, we look for the value of the function when $x = 0, f(0) = 11$, and $g(0) = 8$.

Step 2: Determine which y-intercept is larger. $11 > 8$ and so $f(0) > g(0)$.

Answer: $f(x)$ has a larger y-intercept.

1. Which function has the steeper slope? Explain your response.

$f(x) = 3x - 8$

x	$g(x)$
–2	1
–1	3
0	5
1	7
2	9
3	11

2. Which function has the lower rate of change over the given interval? Explain your response.

$f(x) = 3x^2 - 2x + 5,$ [3, 8]

x	$g(x)$
3	16
4	27
5	42
6	61
7	84
8	111

3. Which functions has the higher y-intercept? Explain your response

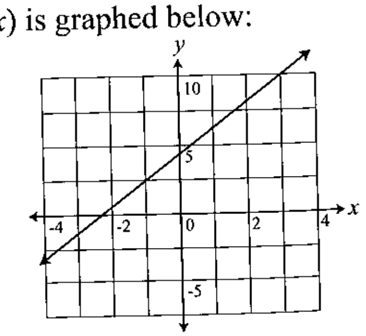
$f(x) = x^3 + 3x^2 + 8$

x	$g(x)$
–2	7
–1	6
0	3
1	–2
2	–9
3	–18

4. Which linear function has the smaller slope? Explain your response.

$f(x) = x + 7.2$

$g(x)$ is graphed below:

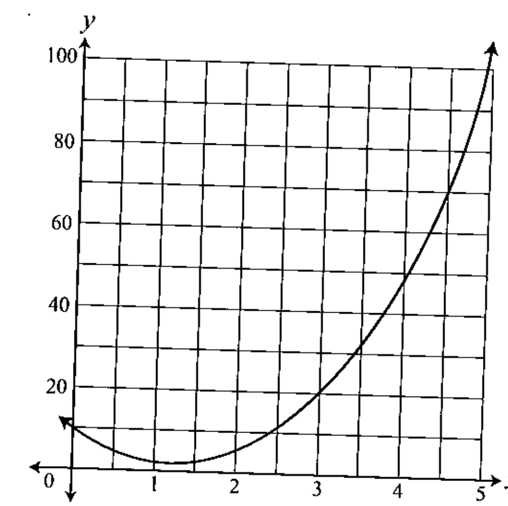


5. Which function has the greater rate of change over the given interval?

Explain your response.

$f(x) = \dfrac{x^2 - 9}{x + 1}, [0, 5]$

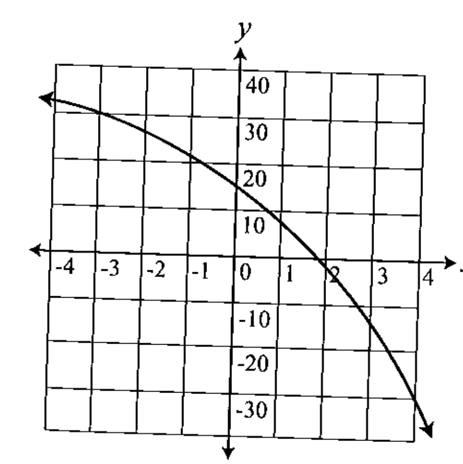
$g(x)$ is graphed below:



6. Which function has the greater maximum? Explain your response.

$f(x) = -x^2 + 17x + 7$

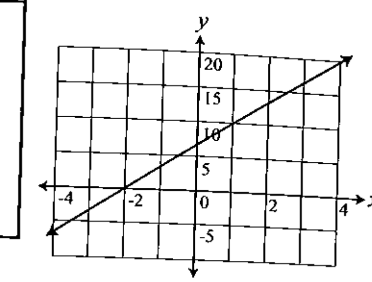
$g(x)$ is graphed below:



7. Which linear function has the steeper slope? Explain your response.

x	$f(x)$
–2	–4
–1	1
0	6
1	11
2	16
3	21

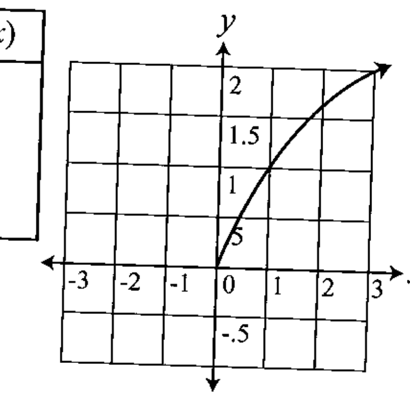
$g(x)$ is graphed below:



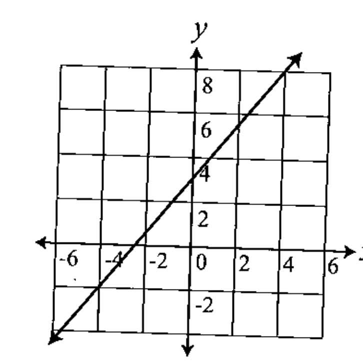
8. Which of the following functions has a smaller rate of change over the interval [0, 3]? Explain your response.

$g(x)$ is graphed below:

x	$f(x)$
0	0
1	1
2	4
3	9



9. Which linear function has the greater y-intercept? Explain your response. $g(x)$ is graphed to the left:



x	$f(x)$
–2	6
–1	7.5
0	9
1	10.5
2	12
3	13.5

10. Which linear function has the greater slope?

$f(x)$ is a function that represents the price of a pizza depending on the number of toppings. Each topping costs \$0.50.

$g(x) = 1.5x + 10.5$

11. Which function has the greater rate of change over the given interval?

$f(x)$ is a function that represents your decision to invest money into a savings account. You initially deposit \$500 and after 5 years, the account holds \$800.

$g(x) = -x^2 + 7x + 6; [0, 5]$

12. Which function has the lower minimum?

$f(x)$ is a function that represents the dimensions of rectangular flags. The dimensions have a 2:1 ratio.

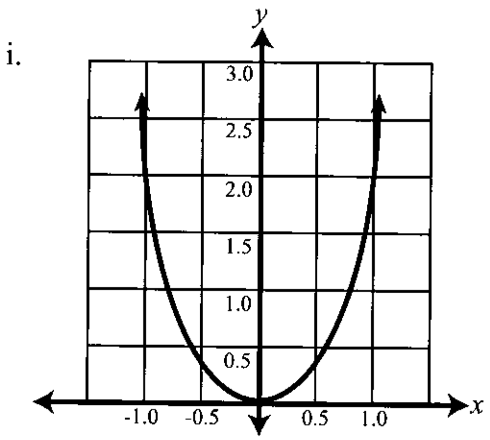
$g(x) = x^2 + 9x + 18$

16.5 Comparing Real-Life Functions (DOK 3)

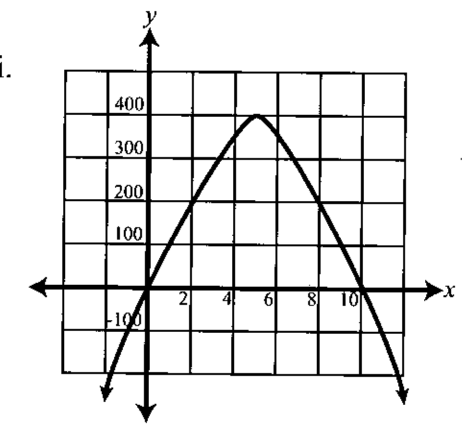
Match the following descriptions of the functions with their algebraic representation, their graph, and their table of values. The graphs and tables are on the following two pages.

1. A cheese pizza is worth \$5.00 and every topping after that is an additional \$0.25.	(a) $a(x) = 800(1.08^x)$
2. A savings account is opened with a deposit of \$800. The interest rate is 8% compounded annually.	(b) $b(x) = \frac{4}{3}\pi x^3$
3. The growth of bacteria quadruples every hour.	(c) $c(x) = -16x^2 + 160x$
4. A toy rocket is launched upward from the ground with an initial velocity of 160 feet per second. Keep in mind that gravity has a velocity of –16 feet per second squared.	(d) $d(x) = 0.25x + 5$
5. In a tournament, 64 teams compete for a trophy. The number of teams is divided by 2 at the end of each round.	(e) $e(x) = 2x^2$
6. The volume of a sphere is a function of the radius of the sphere.	(f) $f(x) = 4e^x$
7. An airplane appears to be descending at a steady rate of 10 feet per minute from 30,000 feet in the air.	(g) $g(x) = 64\left(\frac{1}{2}\right)^x$
8. A woodworker makes wood panels with dimensions in a 2:1 ratio. The area of the flag is a function of the width of the flag.	(h) $h(x) = 30{,}000 - 10x$

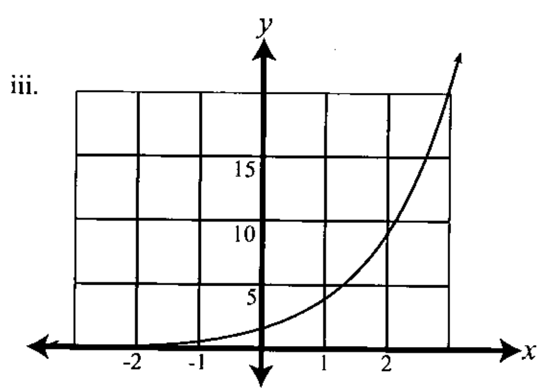
i.



ii.



iii.



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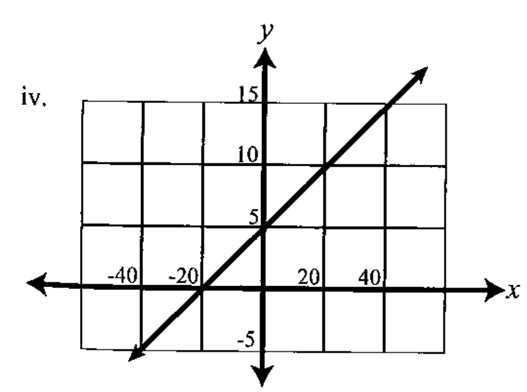


A

x	y
0	0
1	144
2	256
3	336
4	384
5	400

B

x	y
0	1
1	4
2	16
3	64
4	256
5	1024

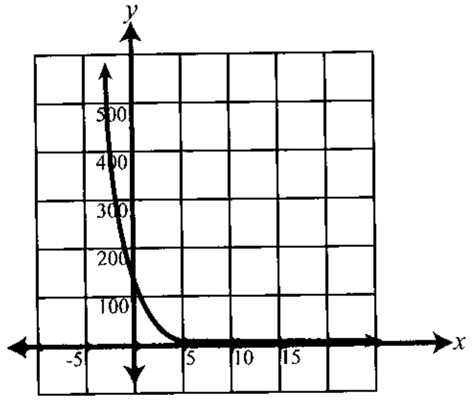
C

x	y
0	800
1	864
2	933.12
3	1007.8
4	1088.4
5	1175.5

D

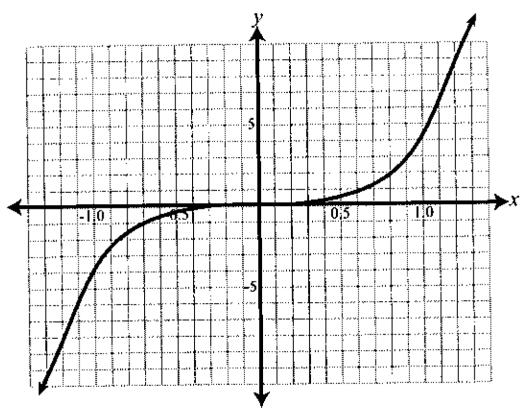
x	y
0	30000
1	29990
2	29980
3	29970
4	29960
5	29950

v.



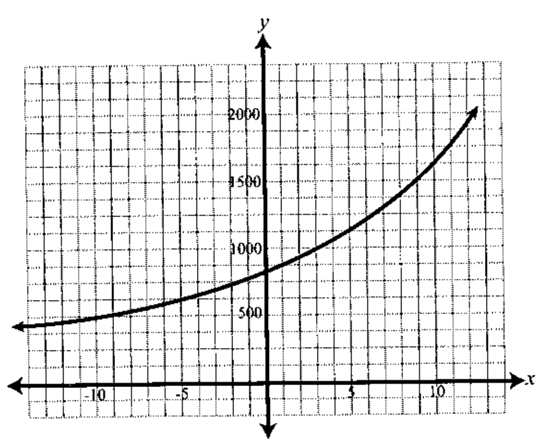
	x	y
E	0	0
	1	4.189
	2	33.51
	3	113.1
	4	268.08
	5	523.6

vi.



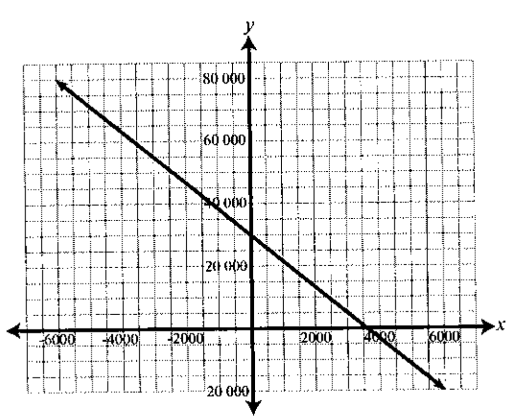
	x	y
F	0	5
	1	5.25
	2	5.5
	3	5.75
	4	6
	5	6.25

vii.



	x	y
G	0	64
	1	32
	2	16
	3	8
	4	4
	5	2

viii.



	x	y
H	0	0
	1	2
	2	8
	3	18
	4	32
	5	50

16.6 Combining Functions (DOK 2)

There are 5 ways to combine functions: addition, subtraction, multiplication, division, and composition.

Example 4: Given $f(x) = x^2 + 6x + 9$ and $g(x) = 4x + 7$ evaluate $(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, $\left(\frac{f}{g}\right)(x)$, and $(f \circ g)(x)$

Step 1: $(f + g) = x^2 + 6x + 9 + 4x + 7 = x^2 + 10x + 16$

Step 2: $(f - g) = x^2 + 6x + 9 - (4x + 7) = x^2 + 2x + 2$

Step 3: $(fg)(x) = (x^2 + 6x + 9)(4x + 7) = 4x^3 + 31x^2 + 78x + 63$

Step 4: $\left(\frac{f}{g}\right)(x) = \frac{x^2 + 6x + 9}{4x + 7}$

Step 5: $(f \circ g)(x) = (4x+7)^2 + 6(4x+7) + 9 = 16x^2 + 56x + 49 + 24x + 42 + 9 = 16x^2 + 80x + 100$

Example 5: Boon works 40 hours a week at a furniture store. He makes $300 weekly salary, plus a 4% commission on sales over $2,500. Assume Boon sells enough this week to get the commission. Given the functions $f(x) = 0.04x$ and $g(x) = x - 2500$. What combination of these two functions represents Boon's commission?

Answer: The composition of these two functions represents Boon's commission. $f(g(x)) = 0.04\,(x - 2500)$.

Problems.

1. The number n of cellphones produced by some factory in one day after t hours is given by $n = 900t - 9t^2$. If the cost C in dollars of producing n cellphones is $C(n) = 11000 + 350n$. What is the cost C as a function of time t?

2. The price p of fish can be purchased at the market per pound f according to the equation $p = 2f + 18$. The amount of tax t due per pound of fish f can be calculated by $t = 0.03f + 1$. What is the total amount (A) a customer would pay for f pounds of fish?

3. A rental car company charges $100 per day, represented by the function $d = 100$. There is an additional charge of $0.45 per mile travelled, represented by the function $r = 0.45m$. What is the function that represents the total cost C of renting a car for one day?

4. The price p of a camera and the quantity x sold are represented by the equation $p = -x + 12$ and the cost C of producing x units is $C = \frac{x^2 + 1000}{15}$. Find the cost C as a function of price p.

5. Your investment s into your savings account is \$800 such that $s = 800$. The savings account has an interest rate r of 9%, compounded annually, represented by the function $r = 1.09^t$. Find the total amount B in the account as a function of time t.

6. A discount d of 20% on price p, represented by the function $d = 0.2p$ is offered on certain products in a store. Assuming someone purchases one of these items, and the full price f is represented by the function $f = p$, what is the function that represents the amount A the customer would pay for this item?

7. The surface area S of a spherical hot-air balloon is given by $S(r) = 4\pi r^2$ where r is the radius of the balloon. If the radius r increases with time t according to the formula $r = \frac{1}{2}t^3$. Find the surface area S of the balloon as a function of time t.

8. The total profit t made on all the tickets to attend a movie can be determined using the function $t = 8.25p - 1$ where p represents the number of people buying tickets. The total number of tickets m sold is represented by the function $m = p$. What is the function that represents the profit R made on each individual ticket?

16.7 Function Symmetry (DOK 2)

In Mathematics, functions can be defined as symmetrical with respect to the y-axis or the origin. To test equations for symmetry, it is helpful to remember the following:

$f(-x) = f(x)$ means the function is symmetrical with respect to the y-axis.
Being symmetrical with the y-axis means the function is **even**.

$f(-x) = -f(x)$ means the function is symmetrical with respect to the origin.
Being symmetrical with the origin means the function is **odd**.

$f(-x) \neq -f(x)$ or $f(x)$ then the function is not symmetrical.

Example 6: Test the following function for symmetry. $f(x) = x^4 + x^2 + 3$

Step 1: First, we need to substitute $-x$ in the function for x.
$f(-x) = (-x)^4 + (-x)^2 + 3$

Step 2: Carry out the operations.
$f(-x) = x^4 + x^2 + 3$

Step 3: Since $f(-x) = x^4 + x^2 + 3$ and $f(x) = x^4 + x^2 + 3$, then $f(-x) = f(x)$.
This means $f(x) = x^4 + x^2 + 3$ is symmetrical with respect to the y-axis.
The function $f(x) = x^4 + x^2 + 3$ is even.