Honors Math 3: **Building Polynomials From Their Roots** Name: Key

1. I am a cubic with roots at -2, -3, and 1. (2, 40) is a point on me. Write my function in **factored form**

$y = a(x+2)(x+3)(x-1)$
$40 = a(2+2)(2+3)(2-1)$
$40 = a(4)(5)(1)$
$40 = 20a$
$2 = a$

→ $y = 2(x+2)(x+3)(x-1)$

2. I'm a cubic with a double root at -4 and a root at 2. The point (0, 96) is on me. Write my function in **standard form.**

$y = a(x+4)^2(x-2)$
$96 = a(0+4)^2(0-2)$
$96 = a(4)^2(-2)$
$96 = -32a \rightarrow a = -3$

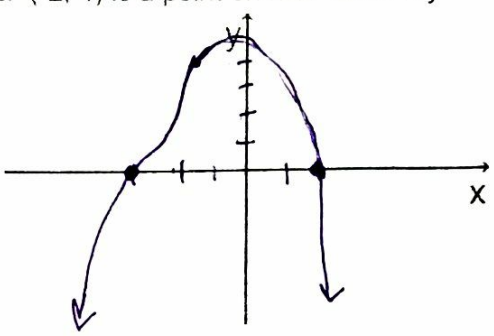
→ $y = -3(x+4)^2(x-2)$
$y = -3(x^2+8x+16)(x-2)$
$y = -3(x^3+6x^2-32) \rightarrow y = -3x^3 - 18x^2 + 96$

$x^2+8x+16$		
x	x^3	$8x^2$	$16x$
-2	$-2x^2$	$-16x$	-32

3. I'm a quartic with a triple root at -3 and a root at 2. (-2, 4) is a point on me. Write my function in **factored form** and sketch me.

$y = a(x+3)^3(x-2)$
$4 = a(-2+3)^3(-2-2)$
$4 = a(1)^3(-4)$
$4 = -4a$
$-1 = a$
$y = -1(x+3)^3(x-2)$

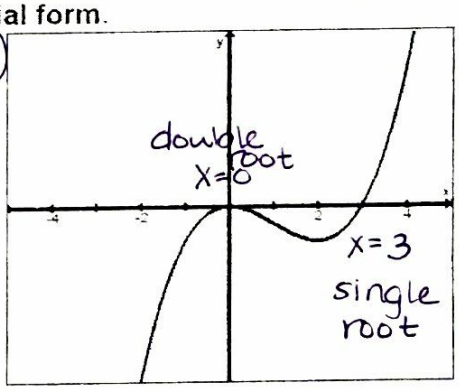
→ down & down



4. f(x) has a minimum of -4 at x = 2. Write in **polynomial form.**

(2, -4)

$y = ax^2(x-3)$
$-4 = a(2)^2(2-3)$
$-4 = a(4)(-1)$
$-4 = -4a$
$1 = a$
$y = x^2(x-3) \rightarrow y = x^3 - 3x^2$



5. In the problems below, I provided you with some of the roots of a polynomial. List any additional roots based on the information provided.

A) $-5, i, -i$

B) $-3+\sqrt{5}, -i$
$-3-\sqrt{5}, i$

C) $-1+i, \sqrt{5}$
$-1-i, -\sqrt{5}$

D) $2, -2+\sqrt{10}$
$-2-\sqrt{10}$

In questions 6-9, write a polynomial with the given roots.

6. $-5,\ \sqrt{3},\ -\sqrt{3}$

$y = (x+5)(x^2-3)$

	x^2	-3
x	x^3	$-3x$
$+5$	$5x^2$	-15

$y = x^3+5x^2-3x-15$

$x = \pm\sqrt{3}$

$x^2 = 3$

$x^2-3=0$

7. $-1,\ 2i,\ -2i$

$y=(x+1)(x^2+4)$

$y = x^3+x^2+4x+4$

	x^2	$+4$
x	x^3	$4x$
$+1$	x^2	4

$x = \pm 2i$

$x^2 = 4i^2$

$x^2 = -4$

$x^2+4=0$

8. $2i,\ -2i,\ 2+2i,\ 2-2i$

$y = (x^2+4)(x^2-4x+8)$

	x^4	$-4x^3$	$8x^2$
x^2	x^4	$-4x^3$	$8x^2$
4	$4x^2$	$-16x$	32

$y = x^4-4x^3+12x^2-16x+32$

$x = \pm 2i$

$x^2 = 4i^2$

$x^2 = -4$

$x^2+4=0$

$x = 2 \pm 2i$

$x-2 = \pm 2i$

$(x-2)^2 = 4i^2$

$x^2-4x+4 = -4$

$x^2-4x+8=0$

9. $2i,\ -2i,\ 2+2\sqrt{2},\ 2-2\sqrt{2}$

$y = (x^2+4)(x^2-4x-4)$

	x^4	$-4x^3$	$-4x^2$
x^2	x^4	$-4x^3$	$-4x^2$
$+4$	$4x^2$	$-16x$	-16

$y = x^4-4x^3-16x-16$

$x = \pm 2i$

$x^2 = 4i^2$

$x^2 = -4$

$x^2+4=0$

$x = 2 \pm 2\sqrt{2}$

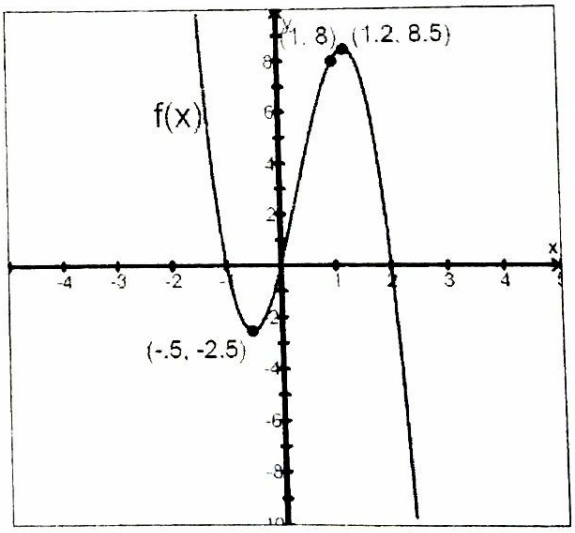
$x-2 = \pm 2\sqrt{2}$

$(x-2)^2 = 8$

$x^2-4x+4 = 8$

$x^2-4x-4=0$

10. Answer the following questions for f(x).



a) Domain: $(-\infty, \infty)$

b) Range: $(-\infty, \infty)$

c) Write the function $y = -4x(x+1)(x-2)$

d) What is f(3)? $= -48$

e) Where is f(x) = 0? $\{-1, 0, 2\}$

f) Where do the maximum(s) occur, if any? $(1.2, 8.5)$

g) Where do the minimum(s) occur, if any? $(-.5, -2.5)$

h) What are the minimum value(s)? -2.5

i) Give interval(s) where f(x) is increasing. $(-.5, 1.2)$

$y = ax(x+1)(x-2)$

$8 = a(1)(1+1)(1-2)$

$8 = a(1)(2)(-1)$

$8 = -2a$

$-4 = a$

$f(3) = -4(3)(3+1)(3-2)$

$= -4(3)(4)(1)$

$= -48$