Name Key Date________ Block______

Point of Concurrency Worksheet

Give the name the point of concurrency for each of the following.

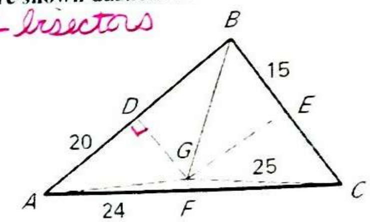
1. Angle Bisectors of a Triangle incenter
2. Medians of a Triangle centroid
3. Altitudes of a Triangle orthocenter
4. Perpendicular Bisectors of a Triangle circumcenter

Complete each of the following statements.

5. The ***incenter*** of a triangle is equidistant from the sides of the triangle.
6. The ***circumcenter*** of a triangle is equidistant from the vertices of the triangle.
7. The ***centroid*** is 2/3 of the distance from each vertex to the midpoint of the opposite side.
8. To ***inscribe*** a circle about a triangle, you use the angle bisectors
9. To ***circumscribe*** a circle about a triangle, you use the perpendicular bisectors
10. **Complete the following chart. Write if the point of concurrency is *inside*, *outside*, or *on the triangle*.**

	Acute Δ	Obtuse Δ	Right Δ
Circumcenter	inside	outside	on (midpt. of hypotenuse)
Incenter	inside	inside	inside
Centroid	inside	inside	inside
Orthocenter	inside	outside	on (vertex of right ∠)

In the diagram, the perpendicular bisectors (shown with dashed segments) of ΔABC meet at point G--the *circumcenter*. and are shown dashed. Find the indicated measure. ↳ from ⊥ bisectors

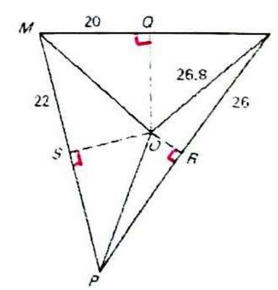


11. AG = 25
12. BD = 20
13. CF = 24
14. AB = 40
15. CE = 15
16. AC = 48
17. m∠ADG = 90°
18. IF BG = (2x – 15), find x.

$2x - 15 = 25$
$2x = 40$
$x = 20$

x = 20

In the diagram, the perpendicular bisectors (shown with dashed segments) of ΔMNP meet at point O—the *circumcenter*. Find the indicated measure.



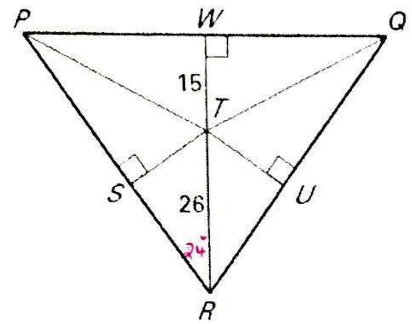
19. MO = 26.8
20. PR = 26
21. MN = 40
22. SP = 22
23. m∠MQO = 90°
24. If OP = 2x, find x.

$2x = 26.8$
$x = 13.4$

x = 13.4

Point *T* is the *incenter* of ΔPQR.

≅ from ∠ bisectors



25. If Point T is the ***incenter***, then Point T is the point of concurrency of

the angle bisectors

26. ST = 15

27. If TU = (2x – 1), find x.

$2x - 1 = 15$

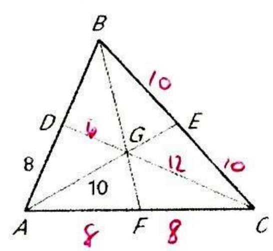
$2x = 16$

x = 8

28. If m∠PRT = 24°, then m∠QRT = 24°

29. If m∠RPQ = 62°, then m∠RPT = 31°

***Point G* is the *centroid* of ΔABC, *AD* = 8, *AG* = 10, BE = 10, AC = 16 and *CD* = 18. Find the length of each segment.**



30. If Point G is the ***centroid***, then Point T is the point of concurrency of

the medians.

31. DB = 8

32. EA = 15

33. CG = 12

34. BA = 16

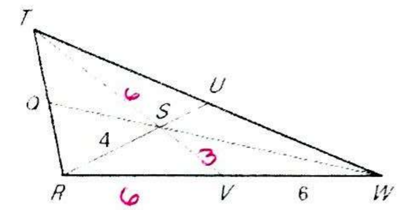
35. GE = 5

36. GD = 6

37. BC = 20

38. AF = 8

***Point S* is the *centroid* of ΔRTW, *RS* = 4, *VW* = 6, and *TV* = 9. Find the length of each segment.**



39. RV = 6

40. SU = 2

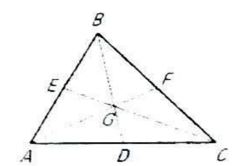
41. RU = 6

42. RW = 12

43. TS = 6

44. SV = 3

Point *G* is the centroid of ΔABC. Use the given information to find the value of the variable.



45. FG = x + 8 and GA = 6x – 4

$6x - 4 = 2(x + 8)$

$6x - 4 = 2x + 16$

$4x = 20$

x = 5

46. If CG = 3y + 7 and CE = 6y

$3y + 7 = \frac{2}{3}(6y)$

$3y + 7 = 4y$

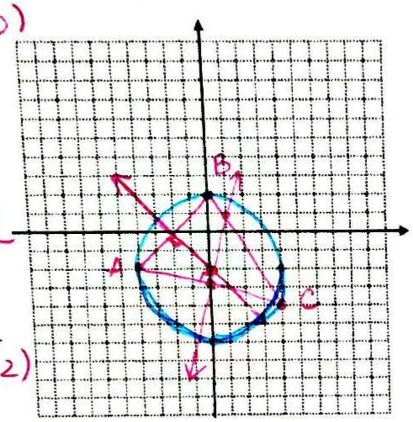
$7 = y$

y = 7

Calculating points of Concurrency: What's the point?

Circumcenter

1. Plot the points $A(-4,-2), B(0,2), C(4,-4)$. Draw ΔABC.



a. Calculate the midpoint of $\overline{AB}$. $(-2,0)$

$\frac{-4+0}{2} = -2, \quad \frac{-2+2}{2} = 0$

b. Find the slope of $\overline{AB}$ $m = 1$

$\frac{2--2}{0--4} = \frac{4}{4} = 1$

c. What is the slope of the perpendicular bisector of $\overline{AB}$ opp reciprical

$m = -1$

d. Write and graph the equation of the line containing perpendicular bisector of $\overline{AB}$.

$y - 0 = -1(x - -2)$

$y = -x - 2$

e. Follow the same steps to write and graph the equation of the line containing perpendicular bisector of $\overline{AC}$.

4a. mdpt: $\frac{4+-4}{2} = 0 \quad \frac{-2+-4}{2} = \frac{-6}{2} = -3 \quad (0,-3)$

4b. slope: $\frac{-4--2}{4--4} = \frac{-2}{8} = -1/4$

4c. $\perp$ slope: $m = 4$

d. $y + 3 = 4(x - 0)$

$y + 3 = 4x$

$y = 4x - 3$

f. Calculate the circumcenter (point D) of ΔABC. Plot and label the point on the graph. Check by constructing a circumscribed circle.

$y = -x - 2 \qquad y = 4x - 3$

$-x - 2 = 4x - 3$

$1 = 5x$

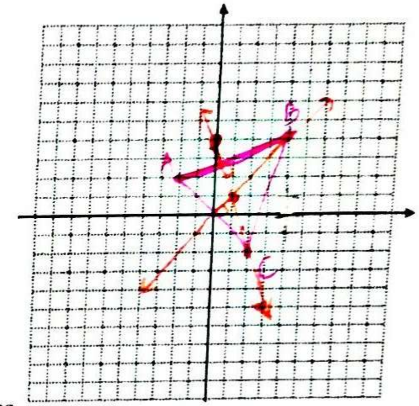
$\frac{1}{5} = x$

$y = -\frac{1}{5} - 2 = -\frac{11}{5}$

Circumcenter $\left(\frac{1}{5}, -\frac{11}{5}\right)$

Orthocenter

2. Plot the points $A(-2,2), B(4,4), C(2,-2)$. Draw triangle ΔABC.



a. Find the slope of $\overline{AB}$. $m = 1/3$

$\frac{4-2}{4--2} = \frac{2}{6} = \frac{1}{3}$

b. What is the slope of altitude $\overline{CD}$? opp. reciprocal.

$m = -3$

c. Write and graph the equation of the line containing altitude $\overline{CD}$.

$y + 2 = -3(x - 2)$

$y + 2 = -3x + 6$

$y = -3x + 4$

d. Follow the same steps to write and graph the equation of the line containing altitude $\overline{BE}$.

$m_{AC} = \frac{-2-2}{2--2} = \frac{-4}{4} = -1 \qquad$ slope of $_{BE} = 1$

$y - 4 = 1(x - 4)$

$y = x$

e. Calculate the orthocenter (point G) of ΔABC. Plot and label the point on the graph.

$y = -3x + 4 \qquad y = x \qquad (1,1)$

$x = -3x + 4 \qquad \to y = 1$

$4x = 4$

$x = 1$

f. Find the equation of the line containing altitude $\overline{AF}$. Your orthocenter should be on that line as well. Substitute your point into this equation to check that it works.

$m_{BC} = \left(\frac{-2-4}{2-4}\right) = \frac{-6}{-2} = 3 \qquad$ slope of $_{AF} = \frac{-1}{3}$

$y - 2 = -\frac{1}{3}(x + 2)$

$y - 2 = -\frac{1}{3}x - \frac{2}{3}$

$y = -\frac{1}{3}x + \frac{4}{3}$

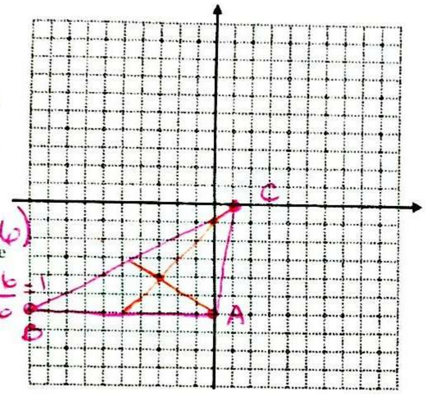
$1 = -\frac{1}{3}(1) + \frac{4}{3}$?

$1 = -\frac{1}{3} + \frac{4}{3}$

$1 \stackrel{\checkmark}{=} \frac{3}{3}$

Centroid

3. Plot the points $A(0,-6), B(-10,-6), C(1,0)$. Draw ΔABC.



a. Find the midpoint of $\overline{AB}$ and label that point D.

$\frac{0+-10}{2} = -5 \quad \Big| \quad \frac{-6+-6}{2} = -6$

$(-5,-6)$

b. Write and graph the equation of the line containing median $\overline{CD}$.

slope of $CD = \frac{-6-0}{-5-1} = \frac{-6}{-6} = 1$

$y - 0 = 1(x-1)$

$y = x - 1$

c. Follow the same steps to write and graph the equation of the line containing the median $\overline{AE}$.

$E = \text{mdpt}_{BC} = \left(\frac{-10+1}{2}, \frac{-6+0}{2}\right) = (-4.5, -3)$

$\text{slope}_{AE} = \frac{-3 - -6}{-4.5-0} = \frac{3}{-4.5} = -\frac{2}{3}$

$y + 6 = -\frac{2}{3}(x - 0)$

$y = -\frac{2}{3}x - 6$

d. Calculate the centroid (point F) of ΔABC. Plot and label the point on the graph. Find the average of your x's and the average of your y's. Do your calculations match?

$y = x - 1 \qquad y = -\frac{2}{3}x - 6$

$x - 1 = -\frac{2}{3}x - 6$

$3x - 3 = -2x - 18$

$5x = -15 \rightarrow x = -3$

$y = -3 - 1 = -4$

$(-3,-4)$

e. Two vertices of a triangle are $(0,0)$ and $(9,0)$. The centroid is $(6,1)$. Find the third vertex of the triangle.

$(9,3)$

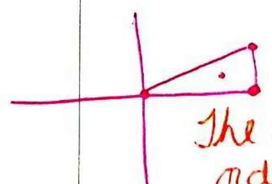
$\frac{0+9+x_3}{3} = 6$

$9 + x_3 = 18$

$x_3 = 9$

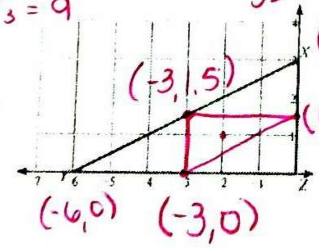
$\frac{0+0+y_3}{3} = 1$

$y_3 = 3$



The centroid is the average of all 3 ordered pairs of the vertices.

f. Connect the midpoints of each side of the triangle to form a smaller triangle within the original triangle. Find the coordinates of the centroid of the smaller triangle. What happened and why?



$\text{mdpt}_{xy} = (-3, 1.5)$

$\left(\frac{-3+0+-3}{3}, \frac{1.5+1.5+0}{3}\right) = (-2, 1)$ it is also the centroid of the larger Δ.